A Schema for Superluminal Communication – What is Wrong?

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Abstract
A modified form of the ESSW experiment is described and a schema of superluminal communication is constructed on it. It is though known that superluminal communication is not allowed by the laws of our universe. The question, what is wrong with this schema, remains open. *Suggestions and comments are invited.*

Abbreviations
ESSW = Englert, Scully, Süssmann, and Walther
QM = quantum mechanics
SR = special theory of relativity

1. Introduction

An apparent schema of superluminal communication is built in base of a modified form of the experiment of Englert, Scully, Süssmann, and Walther (ESSW) [1]. No law of physics seems to be violated in the schema. Though, superluminal communication is known to be impossible, because it implies re-writing the past, as shown by A. Peres [2]. Then, the question remains, what is wrong with this schema?

In continuation, section 2 presents the experiment. Section 3 describes the superluminal communication schema theoretically, and section 4 suggests a technical implementation. Section 5 contains discussions.

2. An ESSW-type experiment

An atom is path-entangled with the polarization of a photon as follows

$$|\Psi(\mathbf{r},t)\rangle = \frac{1}{\sqrt{2}}\left\{\psi_+(\mathbf{r},t)|\mathrm{H}\rangle + \psi_-(\mathbf{r},t)|\mathrm{V}\rangle\right\}, \tag{1}$$

figure 1. ψ_+ is the atom wave-packet taking the upper path, and ψ_- the wave-packet taking the lower path. H and V are horizontal, respectively vertical polarization of the photon. The atom wave-packets are identically polarized, and this polarization is going to play no role in the analysis, from which reason it is not mentioned. Also, the space-time description of the photon evolution is not shown in this equation because in the beginning the photon is described by a single wave-packet, its two polarizations being not space-separated. Later on, it yes is going to be tested, though in the base D/A, diagonal and anti-diagonal.

We work in the 2D geometry. The atom wave-packets are at any time symmetrical at reflection in the axis z = 0, called from now on "symmetry axis",

$$\psi_-(x,z,t) = \psi_+(x,-z,t)\,, \tag{2}$$

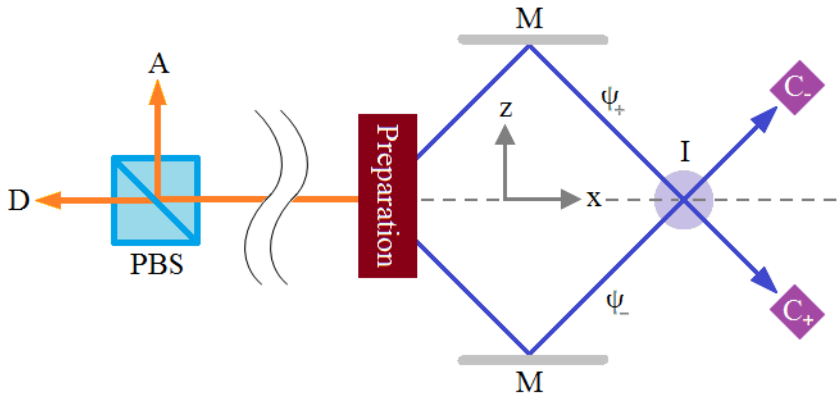


Figure 1. An ESSW-type experiment.
An atom (blue lines) is path-entangled with the polarization of a photon (orange lines) as shows the equation (1) in the text. The two wave-packets of the atom are reflected by the mirrors M and meet in the region I, where at a certain time t^{ov} they overlap exactly. $C_{\pm}$ are detectors. The colors in the figure are only for eye-guiding.

The two wave-packets are reflected by mirrors. The time when the centres of the wave-packets touch the mirrors are taken here as t = 0. In continuation, the wave-packets fly and cross their paths in the region I. At a time labeled here t^{ov} the wave-packets overlap completely.

It is central to this analysis the form taken by the wave-function on the symmetry axis, when the wave-packets meet in the region I, figure 2. Let's pass in the equation (1) from the polarization basis H/V to D/A

$$|\text{H}\rangle = \frac{|\text{D}\rangle + |\text{A}\rangle}{\sqrt{2}}, \qquad |\text{V}\rangle = \frac{|\text{D}\rangle - |\text{A}\rangle}{\sqrt{2}}. \tag{3}$$

On the symmetry axis one gets by virtue of the equalities (2) and (3)

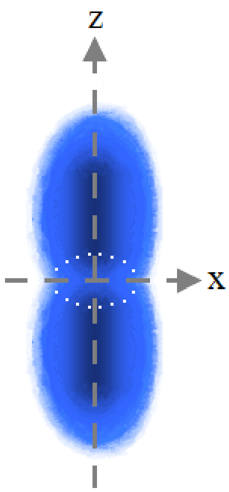
$$|\Psi(x, z=0, t)\rangle = \psi_{+}(x, z=0, t)\frac{|\text{H}\rangle + |\text{V}\rangle}{\sqrt{2}} = \psi_{+}(x, z=0, t)|\text{D}\rangle. \tag{4}$$

However, if on the path of the lower wave-packet is introduced a phase-shift of π

$$|\Psi(x, z=0, t)\rangle = \psi_{+}(x, z=0, t)\frac{|\text{H}\rangle - |\text{V}\rangle}{\sqrt{2}} = \psi_{+}(x, z=0, t)|\text{A}\rangle. \tag{5}$$

3. A schema for superluminal communication

On the manipulation of the atom after passing the mirrors M, is responsible the experimenter Alice. On the test of the photon is responsible the experimenter Bob. Alice and Bob may agree, for instance, that if Bob finds the



photon polarized D, Alice transmitted him a logical "0", but if he finds the photon polarized A, Alice transmitted him a logical "1". On one thing Bob should take care, to test the photon after Alice performs her measurement. For being sure, Bob may do the test after the wave-packets of the atom left completely the region I and are already far from it. The time at which the atom wave-packets are in the region I is known to both experimenters, as the arrival times of the atom at Alice's station and of the photon at Bob's station are synchronized.

As the atom wave-packets pass through the axis z = 0, if one of them contains a particle – if such a thing exists – the particle should cross the axis z = 0 at some time. From the equation (4) it's obvious that if Alice would catch the atom on the axis z = 0, Bob would find the photon polarized D, and if Alice would introduce a phase-shift of π, and then catch the atom on the axis z = 0, Bob would find the photon polarized A.

This would be a superluminal communication schema.

Figure 2. The overlapping of the atom wave-packets in the region I. The white-dotted line in the upper half-plane completes the contour of the lower wave-packet. Analogously, the white-dotted line in the lower half-plane completes the contour of the upper wave-packet.

A couple of problems have though to be solved. One is discussed here below, and the others in the next sections.

The symmetry axis has thickness zero, s.t. it is hard to imagine how an atom can be caught on it. So, in continuation we try to find the thickness of a strip around this axis that Alice can control and still ensure that Bob finds the photon polarized D with higher probability than polarized A. Alternatively, if she inserts the phase-shift of π, Bob should find the photon A polarized with higher probability than D polarized.

We will assume for simplicity that the wave-packets of the atom have Gaussian form in both x and z direction,

$$\psi_{\pm}(x,z,t) = G_X(x,t) G_{\pm}(z,t)\,, \tag{6}$$

$$G_X(x,t) = \frac{1}{\sqrt{\sqrt{\pi}\sigma_x}} \exp\left(\iota \frac{p_{0,x}}{\hbar} x\right) \exp\left(-\iota \frac{p_{0,x}^2 t}{2m\hbar}\right) \exp\left[-\left(\frac{x - p_{0,x} t / m}{2\sigma_x}\right)^2\right], \tag{7}$$

$$G_{\pm}(z,t) = \frac{1}{\sqrt{\sqrt{\pi}\sigma_z}} \exp\left\{\mp \iota \frac{p_{0,z}}{\hbar}[z - u_{\pm}(0)]\right\} \exp\left(-\iota \frac{p_{0,z}^2 t}{2m\hbar}\right) \exp\left\{-\left[\frac{z - u_{\pm}(t)}{2\sigma_z}\right]^2\right\}, \tag{8}$$

see [3], where the functions $u_{\pm}(t)$ give the heights of the centres of the two wave-packets at the time t

$$u_+(t) = a - p_{0,z} t / m = -u_-(t)\,. \tag{9}$$

In particular, at the time t^{ov} the centres coincide and lay on the axis $z = 0$,

$$u_{+}(t^{\mathrm{ov}}) = u_{-}(t^{\mathrm{ov}}) = 0\,. \tag{10}$$

We discuss below the case *without* the phase-shift; the case *with* the phase-shift is completely analogous. By passing in (1) from the polarization basis H/V to the polarization basis D/A – see (3) – then introducing (6), (8) and (9)

$$\left|\Psi(x,z,t)\right\rangle = G_{\mathrm{X}}(x,t)\frac{1}{\sqrt{2\sqrt{\pi}\sigma_z}}\exp\left\{\iota\frac{p_{0,z}}{\hbar}\left[u_{+}(0)-\frac{p_{0,z}t}{2m}\right]\right\}\exp\left[-\frac{z^2+u_{+}^2(t)}{4\sigma_z^2}\right]$$
$$\times\left\{\exp\left(-\iota\frac{p_{0,z}z}{\hbar}\right)\exp\left[\frac{zu_{+}(t)}{2\sigma_z^2}\right]\frac{|\mathrm{D}\rangle+|\mathrm{A}\rangle}{\sqrt{2}}+\exp\left(\iota\frac{p_{0,z}z}{\hbar}\right)\exp\left[-\frac{zu_{+}(t)}{2\sigma_z^2}\right]\frac{|\mathrm{D}\rangle-|\mathrm{A}\rangle}{\sqrt{2}}\right\}. \tag{11}$$

After applying well known trigonometric and hyperbolic formulas, one gets

$$\left|\Psi(x,z,t)\right\rangle = G_{\mathrm{X}}(x,t)\frac{1}{\sqrt{\sqrt{\pi}\sigma_z}}\exp\left\{\iota\frac{p_{0,z}}{\hbar}\left[u_{+}(0)-\frac{p_{0,z}t}{2m}\right]\right\}\exp\left[-\frac{z^2+u_{+}^2(t)}{4\sigma_z^2}\right]$$
$$\times\frac{1}{2}\left\{\left[\mathrm{e}^{-\iota p_{0,z}z/\hbar+zu_{+}(t)/(2\sigma_z^2)}+\mathrm{e}^{\iota p_{0,z}z/\hbar-zu_{+}(t)/(2\sigma_z^2)}\right]|\mathrm{D}\rangle\right.$$
$$\left.+\left[\mathrm{e}^{-\iota p_{0,z}z/\hbar+zu_{+}(t)/(2\sigma_z^2)}-\mathrm{e}^{\iota p_{0,z}z/\hbar-zu_{+}(t)/(2\sigma_z^2)}\right]|\mathrm{A}\rangle\right\}. \tag{12}$$

Thus, the ratio of the probabilities for obtaining the two polarizations of the photon is

$$\frac{\mathrm{Prob}(z,t,\mathrm{A})}{\mathrm{Prob}(z,t,\mathrm{D})}=\frac{\sinh^2[zu_{+}(t)/(2\sigma_z^2)]+\sin^2\left(p_{0,z}z/\hbar\right)}{\sinh^2[zu_{+}(t)/(2\sigma_z^2)]+\cos^2\left(p_{0,z}z/\hbar\right)}. \tag{13}$$

From this relation there results that neither the polarization D, nor A, can be obtained with certainty, unless Alice can test the atom exactly on the axis $z = 0$. As already said, this is not a real possibility. What she can do is to test the atom within a narrow strip centered on the axis $z = 0$, so as to ensure that D appears with bigger probability than A. Such a procedure needs repeated trials, for Bob to gather statistics and notice the difference in probabilities. According to (13) the half-width ζ of this strip is limited by the condition $p_{0,z}\zeta/\hbar < \pi/4$, therefore one gets

$$\zeta < \frac{\pi}{4}\frac{\hbar}{p_{0,z}}. \tag{14}$$

One should also notice another limitation: if the centres of the atom wave-packets are far from the symmetry axis, i.e. $|u_{+}(t)| = |u_{-}(t)| >> \sigma_z$, the values of the trigonometric functions may become negligible in comparison with the hyperbolic sine. In this case the polarizations D and A appear with equal probability. However, in a

Gaussian wave-packet the greatest probability to find the particle is near the centre, which means that the most frequent situations would be that Alice gets a detection at a time close to t^{ov} when u_+ vanishes.

4. A technical suggestion of implementation

The detection of the atom in the neighborhood of the axis $z = 0$ is not a trivial task. Placing a detector, say, with the window looking upwards, disturbs the movement of the wave-packet ψ_-, and placing the detector with the window looking downwards disturbs the movement of the wave-packet ψ_+.

The idea suggested here is to send toward the region I a beam of photons with energy $\hbar\omega$ at least as high as to ionize the atom. The beam should be centered on the axis $z = 0$, with a waist of 2ζ in the region I, where ζ obeys the relation (14). Locating the region I between the plates of a charged capacitor, figure 3, connected to an amplifying circuit, the ion and the electron would be attracted to the capacitor plates and produce a tiny change in the capacitor voltage. This change would be amplified and detected.

Let's get an idea about the order of magnitude of ζ. The calculus below is done on the hydrogen atom, however, for implementing the schema, another type of atom may be found with a higher absorption cross section for the ionizing photons.
It is clear from the inequality (14) that the lower is the atom transversal velocity $v_{0,z}$, the wider may be waist 2ζ of the ionizing beam. For instance, if $v_{0,z} = 1\text{cm/s}$, then

$$2\zeta < \frac{\pi}{2} \frac{\hbar}{M\, v_{0,z}}, \tag{15}$$

where M is the atom mass. There results $2\zeta = 10\mu\text{m}$.

The wider is the transversal waist of the photon beam, the sharper may be its energy, i.e. the imprecision may be lower. The photon energy is given by

$$E = c\sqrt{p_x^2 + p_z^2}, \tag{16}$$

where p_x and p_z are the longitudinal, respectively transversal, linear momenta of the ionizing photon. If $p_z << p_x$, one can approximate $E \approx cp_x + ½\, cp_z(p_z/p_x)$, where from one gets $\Delta E \approx c\Delta p_z(p_z/p_x)$. The uncertainty principle requires $\Delta p_z \geq \hbar / \Delta z$, which in our case means $\Delta p_z \geq \hbar/(2\zeta)$. Thus, one obtains

$$\Delta E \geq \frac{\hbar c}{2\zeta} \frac{p_z}{p_x}. \tag{17}$$

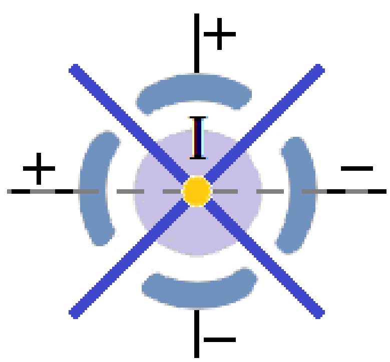


Figure 3. A suggested schema of detection.
The region I is surrounded by the plates of a charged capacitor for attracting the ions. The positive plate is discontinuous there where the atom beam has to pass, and so the negative plate.

Since $p_z << p_x$, for satisfying the above inequality it is sufficient that $\Delta E = \hbar c/(2\zeta)$. With the width of 10μm obtained above, one finds that the uncertainty in energy is negligible, approximately three orders of magnitude lower than the ionization energy of the hydrogen atom.

5. Discussion – what is wrong with this schema?

The laws of our universe don't permit re-writing the past, as proved in [2]. So, something should be wrong. The question is, what is wrong?

The problem of finding a type of atom with high absorption cross section for the ionizing photons does not seem a big impediment, as the probability of absorption is also proportional to the intensity of the photon beam.

Another thing to be questioned is the assumption that the particle crosses at some time the symmetry axis. Indeed, the trajectory of the particle may be not unidirectional, it may have a wild form, and eventually, a particle may not cross the strip 2ζ around the axis z = 0.
However, the distribution of the particles is supposed to obey the QM. Gathering statistics in the strip 2ζ during many trials, and given that the movement of the wave-packets is known, i.e. in each trial and trial the positions u_+ and u_- are known at each time, the probability of getting a detection in this strip is expected to obey the quantum statistics,

$$\mathrm{Prob}(\mathrm{z} < \zeta, t) = ½ \int_{-\zeta}^{\zeta} \mathrm{dz} \left[|G_+(z,t)|^2 + |G_-(z,t)|^2 \right], \tag{18}$$

no matter how queer the trajectory of the particle may be.

One may bring the objection that the ionization of an atom disturbs the wave-function (12). However, from the moment the atom is ionized, the wave-function is no more of interest.
An alternative objection could be that the ionizing beam disturbs the wave-packets in the strip 2ζ invalidating the superposition (12). Indeed, the absorption of a photon increases the atom energy (for this matter it is not relevant if we speak of the atom after absorption or of the ion-electron pair) and changes its linear momentum. Such a perturbation could be seen in a real experiment, i.e. it should not exist a strip 2ζ as wide as allows (15), and in which $\mathrm{Prob}(z,t,\mathrm{A}) < \mathrm{Prob}(z,t,\mathrm{D})$ in the absence of the phase-shift of π. Though, theoretically, the change in linear momentum of the atom is on a direction perpendicular on z, therefore cannot influence the relation (13).

Bottom line, the question what is wrong with this schema, remains open.

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