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The Bravais pendulum: the distinct charm of an almost forgotten experiment

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Abstract

In the year 1851 in Paris, the apparent change of the plane of oscillation of a linear pendulum was observed by Léon Foucault. In the same year, at the same place, the unequal duration of the oscillations of a right- and left-handed conical pendulum was observed by Bravais. Today, the Foucault pendula are common at universities, the Bravais pendula not at all. We have revisited and experimentally tested Bravais's method and found it worthy of revitalization in student labs.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

In 1851, Léon Foucault in Paris raised an idea that a simple linear pendulum, in keeping with the laws of mechanics, should maintain its plane of oscillation in the direction in which it was started. It meant that the oscillating pendulum could supply direct evidence of the axial rotation of the Earth.

Foucault immediately began his promising experiments with a pendulum not more than 2 m long. However, he found the results unsatisfactory. In the second attempt, he increased the length to 11 m, but there were no radical improvements. Finally, under the great dome of the Pantheon, Foucault set up the famous 67 m long pendulum, and positive results were reported [1]. It was the first simple proof of the Earth's rotation in an easy-to-see experiment (without watching for apparent movement of heavenly bodies).

Foucault did not report very high accuracy in his measurements. The pendulum exhibited somewhat unexpected movement, which was beyond the limits of the author's prior expectations. Nevertheless, his contemporaries recognized Foucault's pendulum project as a very good idea. Longden wrote: 'The pendulum experiment of Léon Foucault . . . is held in reverence by physicists and astronomers, not simply as one of the successful experiments of the nineteenth century, but as one of the brilliant experiments of all time' [2].

Soon a wave of building various linear pendula 'à la Foucault' arose in many countries around the world. Out of these numerous pendula the ones which are very long and, furthermore, are in places with a rather large latitude, work plausibly—they are the real attractions that permanently play a considerable pedagogical and tourist role. A good example of this was the Foucault pendulum in Saint Petersburg, exposed in Saint Isaac's Cathedral (pendulum length 98 m; installation removed in the 1990s)[1]. The list of carefully designed Foucault pendula around the world (mainly at universities, museums and planetaria) is rather impressive[2]. Some years ago, *Physics World* published an interesting ranking of physical experiments ever carried out [3]; physicists who were asked to nominate the most beautiful experiment of all time included Foucault's pendulum in the top 10 winners.

Soon after 'the beautiful experiment of Foucault on the deviation of the plane of oscillation' was carried out, Bravais observed that the unequal duration of the vibrations of a right- and left-handed pendulum should be a necessary consequence [4]. To test his idea, Bravais made a pendulum 10 m long with a weight of 10.5 kg. He diligently used it to operate as a conical pendulum sensitive to the Earth's rotation. The results appeared to be quite satisfactory; in the author's own words: 'when the pendulum rotates from west to east, the angular velocity at Paris is retarded $11''.4$ per second of time; on the other hand, when the motion is from east to west, the velocity is increased by the same amount.' What happened afterwards?

The Bravais pendulum differs from the Foucault pendulum: the deviation in azimuth cannot be observed in conical oscillations—there is only a change in the period of oscillation. Now, many people believe it is a simple psycho-physiological fact that a variation of space appeals more to human senses than one of time. The astronomer W F Rigge wrote in his popular review article 'Experimental proofs of the Earth's rotation' [5]: 'The Bravais pendulum never met with popular favor, and seems to have been set up only once, and that by the inventor himself.'

This way or another, the conclusion is rather plain and could be stated as follows: in today's world, for the purposes of detecting the Foucault effect (the direct observation of the Earth's rotation), we see a lot of linear or mathematical pendula, and only few (if any) conical pendula. Foucault's work is celebrated, and Bravais' variant is forgotten.

In order to enlighten this intricate point, we have undertaken a project to revisit Bravais's method and eventually learn more on pro and contra arguments. Here, we report our actions and conclusions.

2. Conical pendulum

We set up a pendulum in the hall of the Faculty of Science in the town of Kragujevac, Republic of Serbia; the latitude of Kragujevac is $\lambda_K = 44^\circ\ 1'$N and the longitude $\delta = 20^\circ\ 55'$ E.

We attached a ball to a string as sketched in figure 1. The effective length of the string (from the point of suspension P on the ceiling to the centre of the leaden ball) is $L = 14.1$ m. The leaden ball ($m = 23$ kg) is 15.7 cm in diameter.

We shall use this pendulum in the conical mode of operation. The ball revolves in a horizontal circle of radius A (just above the floor) with constant speed v_0. Let T_0 be the period of revolution. In the applied inertial frame of reference, two forces govern the behaviour of the physical system: the force $\vec{T}$ exerted by the string, and the weight $m\vec{g}$. The vertical component $T\cos\alpha$ must balance the weight (inset, figure 1). The radial component $T\sin\alpha$ provides centripetal acceleration; hence, we have for the speed $v = \sqrt{Ag\tan\alpha}$. Let us insert

[1] On the occasion of my (VMB) short scientific visit to the Ioffe Institute in 1973, I took advantage of the opportunity to see this *scientific* shrine; I attended a series of demonstrations and found the spectacle impressive.

[2] The Internet site *List of Foucault pendulums.*

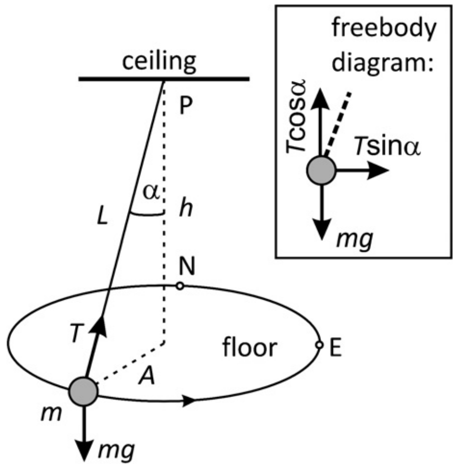

Figure 1. Conical pendulum and corresponding free body diagram.

here $A = L\sin\alpha$. The ball travels the circumference of the circular path in a time equal to the period of the pendulum and we obtain

$$T_0 = 2\pi\sqrt{\frac{h}{g}}, \tag{1}$$

because $L\cos\alpha$ is the height of the pendulum, h.

In our measurements we have strictly worked in the small-angle regime, limiting the maximal radius of the ball orbit to $A_{\max} = 50$ cm. The half-angle of the cone (α) is then 0.035 rad, or 2°. Accordingly, we can assume $T_0 \approx 2\pi\sqrt{L/g}$ and $\omega_0 = \sqrt{g/L}$ (the angular frequency of the pendulum). Thus, our pendulum is actually in the isochronous regime of oscillation. For example, with the chosen initial radius $A = 45$ cm, the computed period is 7.53 s. The radius falls slightly in the course of rotation, and after hundreds of complete revolutions reads $A = 39$ cm (as we observed in the experiments). Nevertheless, the figure of the period stays practically unchanged.

Bravais recommends a special mechanism, a kind of clockwork, to start the pendulum weight around in a circle. We proved this clever method to be effective. However, one can easily practise launching the ball by hand. Let us imagine that one has pushed the ball from point E (the east point of the horizontal circular trajectory, figure 1) normally to the west–east direction. If the gained impulse is right, the ball should cross the direction south–north in the north point N—the conical motion is correctly initiated. In two or three attempts one could realize a satisfactory circular orbit. We do believe that the necessary manipulations in achieving conical oscillations should not be too sophisticated, and convenience is important for everyday demonstrations of the Earth's rotation.

3. Theoretical background

We will now give (1) a brief mathematical treatment of the assumed physical picture and (2) expectations in experiments.

(1) Imagine now that the ball of the conical pendulum is moving over the horizontal plane (floor). The point of suspension of the pendulum is on the z-axis. From the kinematic

point of view one could reason as follows. As part of the Earth's surface this plane rotates with the constant angular velocity

$$\vec{\Omega} = \vec{e}_z \Omega_T \sin\lambda, \tag{2}$$

where Ω_T is the Earth's rotation and λ is the latitude. The pendulum oscillates in the 'absolute space' (inertial frame of reference) with an angular frequency ω_0. This circular motion remains circular too for the observer on the plane (rotating frame of reference). For this observer, however, the apparent frequency of the left-handed pendulum (counter-clockwise (ccw) rotation in the northern hemisphere) is slightly decreased:

$$\omega_- = \omega_0 - \Omega, \tag{3}$$

and for the right-hand pendulum (clockwise (cw) direction of rotation) is slightly increased:

$$\omega_+ = \omega_0 + \Omega. \tag{4}$$

In the first case, the observer moves away from the ball and sees a frequency ω_- that is lower than the 'source' frequency ω_0; he could call it 'slow pendulum mode'. In the second case, the observer approaches the ball and sees a frequency ω_+ that is greater than ω_0; he could call it the 'fast pendulum mode'. These facts motivate us to recognize the phenomenon as the Doppler effect for the rotational motion [6].

Now we shall describe the motion of the pendulum as seen from the non-inertial, rotational reference frame. (The rigorous, detailed treatments of the subject can be found in many textbooks; we recommend [7–9].)

Newton's second law in the rotating frame takes the form

$$m\ddot{\vec{r}} = \sum \vec{F}_{\mathrm{rf}} + \sum \vec{F}_{\mathrm{ff}}. \tag{5}$$

Here m is the mass of a body being acted upon by these forces. The first term represents all relevant real forces (forces from physical interactions). The second term is the sum of fictitious forces that include the Coriolis force, the centrifugal force and the Euler force. In our task, there is no Euler force (the frequency is constant). Further, we shall ignore the terms that are proportional to the square power of the Earth's rotation. The exception is the quadratic term multiplied by the Earth's radius $R_T = 6.378 \times 10^6$ m. However, this component combines with the gravitational acceleration $\vec{g}_0$ to give apparent free fall acceleration $\vec{g}$. So, the equation of motion we must deal with turns out to be

$$m\ddot{\vec{r}} = \vec{T} + m\vec{g} - 2m(\vec{\Omega}_{\mathrm{T}} \times \dot{\vec{r}}). \tag{5$'$}$$

Here, $\vec{T}$ is the force exerted by the pendulum string. Let us introduce Cartesian coordinates with the centre at K, as shown in figure 2; x and y lay in the horizontal plane, z is in the direction of $\vec{g}$—vertically upwards. Suppose the angle between the string and the vertical is small; in that case the approximations $T_x \approx -mgx/L$, $T_y \approx -mgy/L$ and $T_z \approx mg$ are valid. In addition, the component $\dot{z}$ is much smaller than either $\dot{x}$ or $\dot{y}$.

Owing to these assumptions, from the vector equation (5′) it follows that the horizontal coordinates (x, y) of the pendulum satisfy the set of equations

$$\ddot{x} - 2\Omega\dot{y} + \omega_0^2 x = 0, \tag{6}$$

$$\ddot{y} + 2\Omega\dot{x} + \omega_0^2 y = 0. \tag{7}$$

The equations are coupled. Note that one can introduce dimensionless variables x/L, y/L h/L and $t\sqrt{g/L}$, dividing by $\omega_0^2 = g/L$.

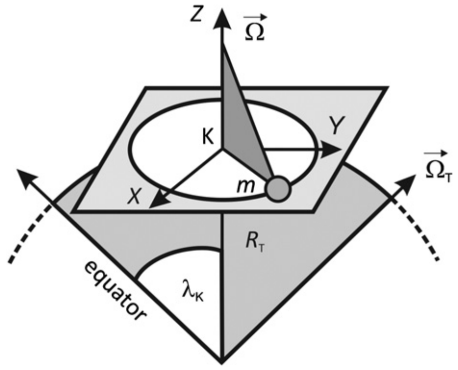

Figure 2. A pendulum in the rotating frame of the Earth.

Now we define the variable $\rho = x + iy$, the complex position of the mass (as recommended, for instance, in [6], [10]). The idea has a simple graphical interpretation: a plot of ρ is a bird's eye view of the pendulum projected onto the horizontal (x, y) plane. From equations (6) and (7) we obtain the equation for the complex position

$$\ddot{\rho} + 2i\Omega\dot{\rho} + \omega_0^2\rho = 0. \tag{8}$$

There are several methods of solving this equation. A popular way is to try a solution of the form $\rho(t) = \exp(i\gamma t)$. This assumption is only valid if we allow $\gamma = \Omega \pm \sqrt{\Omega^2 + \omega_0^2}$. Because Ω^2 (order of magnitude 10^{-10} s^{-1}) is much smaller than ω_0^2 (order of magnitude 10^2 s^{-1}) the approximation $\gamma = \Omega \pm \omega_0$ is quite satisfactory. We obtain the general solution to the pendulum differential equation

$$\rho(t) = A_0 \exp(i\omega_- t) + B_0 \exp(-i\omega_+ t). \tag{9}$$

The solution is composed of two circular polarizations. The parameters A_0 and B_0 are dependent on initial conditions. In fact, the frequencies of pendulum oscillations are modified by the Earth's rotation, as we have anticipated in (3) and (4).

(2) Repeat our basic assumptions: the coordinates (x, y, z) belong to the rotating reference frame and the coordinates (ξ, η, ζ) to an inertial reference frame. The rotation is about the z-axis with an angular velocity Ω. The two reference frames coincide at time $t = 0$. The frames are aligned at $t = 0$.

Choosing the initial conditions $\rho(0) = R$ and $\dot{\rho}(0) = iR\omega_0$, we activate the left-handed pendulum (i.e. launch the circular motion of the ball in the ccw direction, as seen from the ceiling; an almost forgotten idiom is sinistroral revolution). This mode of oscillation has the period T_- (in the Earth's frame of reference). Afterwards, we launch the right-handed pendulum (cw rotation; dextrorsal revolution) according to the conditions $\rho(0) = R$ and $\dot{\rho}(0) = -iR\omega_0$. This opposite mode of oscillation has the period T_+. In the inertial frame of reference (ξ, η) both left- and right-hand polarizations have the same period of rotation $T_0 = 2\pi/\omega_0$. In the Earth's frame of reference (x, y) the two periods are not equal: $T_- > T_0$ and $T_+ < T_0$. The product $\omega_- T_0$ is less than 2π, and $\omega_+ T_0$ is greater than 2π; these facts are illustrated in figure 3 (left-handed pendulum) and figure 4 (right-handed pendulum). The start point on each hodograph is $\rho(0) = R$, and the end point $\rho(T_0) \approx -iR\Omega T_0 = -iR\theta$. (We have abbreviated $\theta = \Omega T_0$.)

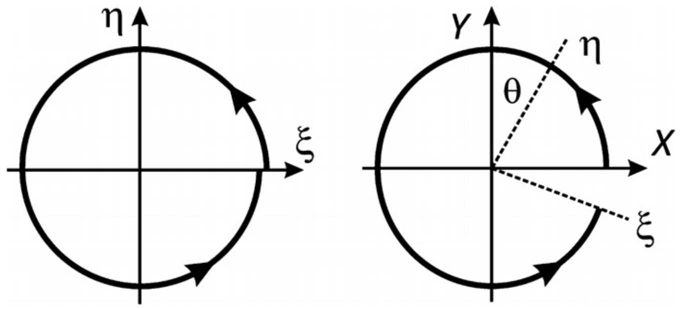

Figure 3. Ball positions in two reference frames in relative motion, for the time interval $(0, T_0)$. 'Slow' mode case.

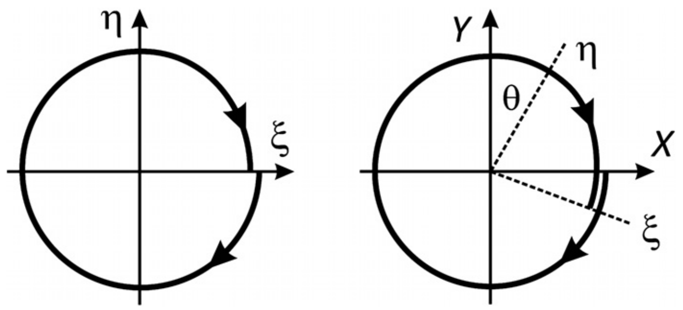

Figure 4. Ball positions in two reference frames in relative motion, for the time interval $(0, T_0)$. 'Fast' mode case.

Evidently, these figures are simple consequences of the fact that $2\Omega = (\omega_+ - \omega_-)$. Straightforwardly we achieve a method of how to infer the Earth's axial rotation from measurable time intervals. Let us compute the period difference ΔT of the two circular motions:

$$\Delta T = T_- - T_+ = \frac{2\pi}{\omega_0 - \Omega} - \frac{2\pi}{\omega_0 + \Omega}. \tag{10}$$

As we have already seen, the approximation $\Omega \ll \omega_0$ is well satisfied, and we obtain

$$\Delta T \approx 2T_0 \frac{\Omega}{\omega_0}, \tag{11}$$

from whence the (local) frequency can be found to be

$$\Omega = \frac{\pi}{T_0^2} \Delta T. \tag{12}$$

Here we shall enter a small digression. The ball moves along a circular path of radius R. Let v_0 be its tangential (linear) velocity. That is, we can use the relation $T_0 = (2\pi R/v_0)$. Note that the quantity πR^2 is equal to the area S of the circle. Consequently, we obtain from equation (12) the alternative formula

$$\Omega = \frac{v_0^2}{4S} \Delta T. \tag{12$'$}$$

If v_0 is replaced by c_0 (velocity of light), we arrive at the conclusion that $4S\Omega = c_0^2 \Delta T$. And this is the basic statement of the equivalent optical Sagnac effect [11].

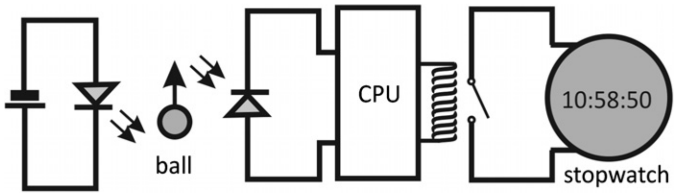

Figure 5. Opto-electronic device for time-interval measuring.

The period of each mode of rotation is a measurable quantity; thus, the time interval ΔT can be immediately calculated from the experimental data. The parameter T_0 is known from the expression $T_0 = 2\pi\sqrt{h/g}$, or at least as the approximation $T_0 \approx \sqrt{T_- T_+}$.

So, equation (12) offers a good base for the determination of the Earth's rotation. Now we shall describe and analyse our tests of Bravais's method.

4. Chronometer

Surely, modern electronic chronometers give us an advantage over the time-measuring devices of Bravais's age. We have realized a time-measuring system according to the scheme shown in figure 5.

First, let us remark that contemporary mobile phone stopwatch software provides resolutions of 1/100 s, or even better. The problem occurs when one tries to start and stop the stopwatch manually—a reflex action is needed to create a movement in response to a stimulus. Reaction time (RT) is the time from the onset of a stimulus until the organism responds. For example, we have to press a button as soon as a light appears. So-called simple RT is the time in which we respond to the presence of the visual stimulus. Mean RT for college-age individuals is about 190 ms to detect visual stimulus. This process could introduce impermissible errors for our purposes. However, an experimenter has a chance, because of the random nature of RT, to obtain measured time intervals of tolerable uncertainties. Our measurements show that in 70 cases out of 100 attempts, a mobile phone stopwatch gives the right sign for the Earth's rotation, and in 40 cases results are with errors never greater than 15%.

Second, we will give some details about our chronometer. We have assembled a time-measuring system according to the scheme in figure 5. In fact, we use a standard stopwatch which can be found in every student lab. It is a sufficiently precise instrument for our purpose. All we have to change is to improve its start and stop performances. Our simple automatic system removes possible uncertainties caused by (unsteady) human reflexes. The electronic circuit is tuned to count the number of ball revolutions and to command a small relay to start and stop the stopwatch counter. The digital stopwatch is directly connected to the relay contacts.

In the heart of the electronic circuit, we place the micro chip (AT MEGA8) which could be programmed for specific tasks. It communicates with external devices via a number of input and output ports. One of the input port pins is connected to the photo couple located across the ball path. Each time a spike at the bottom of the ball intersects the optical beam between two optoelectronic elements, an impulse is generated and transmitted to the control chip. The command coil of relay is connected to the output pins. The CPU sends signals through the coil to start and stop the stopwatch counter.

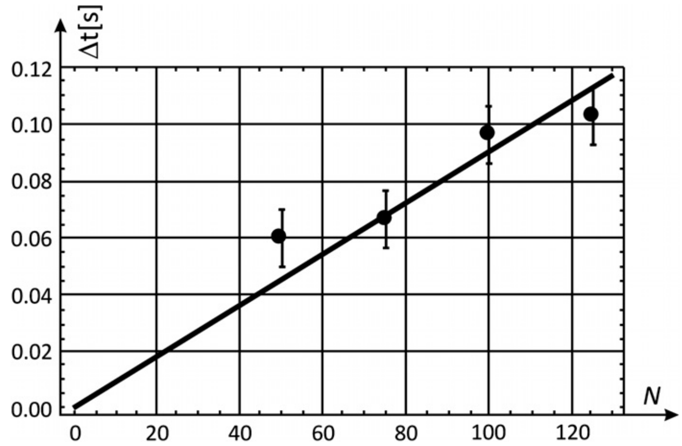

Figure 6. Experimental results: time difference versus number of pendulum turns.

We proved that the device works reliably; it precisely gives the elapsed times t_+ and t_- for any prescribed number of ball revolutions N in advance.

5. Measurements and results

We have carried out four series of measurements, for four total number of ball revolutions: $N_1 = 50$, $N_2 = 75$, $N_3 = 100$ and $N_4 = 125$. For each N_i, the chronometer registers t_- (the time elapsed in the right-hand mode of operation) and t_+ (the time elapsed in the left-hand mode of operation), and afterwards we compute the time difference $\Delta t = t_+ - t_-$. Actually, we always repeated the two procedures 10 times, finally finding the mean value $\Delta t = 0.1 \cdot \sum_1^{10} \Delta t_{\mathrm{i}}$. The set of discrete events of measured time intervals Δt_{i} tend to cluster around the mean value; the distribution function resembles a normal distribution.

Figure 6 summarizes our results. The straight line $\Delta t = qN$ represents the best fit of experimental points if the coefficient q equals 0.9 ms. Uncertainty bars show the $2\sigma^2$ limits (σ^2 is the standard deviation).

According to equation (12), the Earth's angular velocity deduced from the experiments with a conical pendulum in Bravais's sense is

$$\Omega_B = \frac{1}{4\pi} \frac{q}{\sin\lambda} \frac{g}{h}. \tag{13}$$

The latitude of Kragujevac is $\lambda_K = 44^\circ\ 1'$ and therefore $\sin\lambda_K = 0.6946$. Inserting $g = 9.81$ m s^{-2}, $q = 0.0009$ s, as well as the pendulum height $h = 14.09$ m, we get $\Omega_B = 7.18 \times 10^{-5}$ rad s^{-1}. So, our result is only 1.5% smaller than the standard sidereal rate of rotation of our planet $\Omega_T = 7.29 \times 10^{-5}$ rad s^{-1}, and could be understood as a quantitative proof of the Earth's rotation.

6. Comments

To plan, make and use the Foucault pendulum is not an easy job. Experimenters usually encounter two major difficulties: imperfections in the suspension and imperfections of initial conditions. These imperfections, independently or together, quickly develop an oval motion of the ball [12]. Ellipticity induced from the outside, as well as from the inside, efficiently masks the pendulum precession due to the Earth's rotation [2]. The rate of rotation (in degrees per h) of the polarization plane of the *real* Foucault pendulum follows the expression

$$\Omega_F = 15\left[1 - \tfrac{3}{8}\beta^2\right]\sin\lambda. \tag{14}$$

Here β is the oscillation amplitude of the pendulum ball divided by the string length, $\beta = A/L$. The length given, short amplitudes diminish the undesirable second term in brackets (the limit $\beta \to 0$ leads to the ideal pendulum). However, long amplitudes provide better measurements of angles. A compromise in β is needed. The lack of tall rooms inspired some physicists to build short [13] and even very short [14] pendula. Such setups require pendulum path corrections. One could regret this, but any new electronic or mechanical element in the setup which directly affects the free motion of the ball deteriorates the beauty of the demonstration; sometimes, maybe the final result could no longer be called obvious.

The Bravais pendula have, as the experts already stated [5], two points of superiority over the Foucault pendula: (a) it is reversible—may be given a dextrorsal or sinistrorsal revolution, and (b) the time of revolution may be lengthened or shortened at will.

In our measurements we have also found this flexibility of Bravais's method quite practical. Usually, no specific problems are encountered during the experiments. The students claim that the time-interval measurements are not less attractive than those of angles. With some amusement they note that a mobile telephone chronometer secures qualitative proof of the Earth's diurnal rotation, in the sense that the left-handed rotation (statistically convincing) lasts longer than the coupled right-handed rotation. In addition, the analogies with some of the physical effects in other branches of physics [12, 15] are pedagogically valuable.

Bravais himself designed the pendulum according to his own idea, with carefully conducted measurements[3], and finally wrote the paper in a somewhat laconic manner but with logical rigour. However, at a point in the past, and we don't know how, the Bravais pendulum seems to have lost its good reputation. We believe that Bravais's sane idea is worthy of revitalization.

We are sure the Bravais pendulum can enrich many student laboratories, at least as a mechanics lab project. It could also be an elegant demonstration experiment installation in museums and planetaria.

Acknowledgments

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[3] Bravais wrote: 'M Arago, having had the goodness to place the great meridian room of the observatory at my disposal, I was able to make the experiments necessary to the testing of this point, with a pendulum 10 metres long; and thus not only to confirm the predictions of theory, but to do it with a degree of accuracy which I had not previously ventured to hope for.'

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