Klausur: 41050 Mathematical Methods I Winter term 2017/18
Prüfer: apl. Prof. Dr. F. Werner

Working time: 60 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient. It is not allowed to use mobile phones or smart watches.

Tools:
- pocket calculator (according to the instructions of FWW)
- **either** one individually prepared one-sided A4 sheet of paper with arbitrary material (write '1' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet). If the formula sheet is used, please add your name and matriculation number and hand it in together with your examination.

Problems:

1. (a) Determine all real solutions of the equation

$$\log_a\left(\frac{a^3}{4}\right) + \frac{1}{2}\log_a(2x+3) = \frac{1}{2}\log_a\left(x+\frac{1}{2}\right) + a^{\log_a 3}.$$

(b) A company produces 15,000 cars in the year 2001. The production increases every year by 6 %. What will be the first year with more than 45,000 produced cars per year, and what is the total production within the period 2001- 2010?

(c) Determine all real zeroes of the polynomial $P_3(x) = x^5 + x^3 - 12x$ and write P_3 as a product of linear and quadratic terms.

(20 points)

2. (a) Given is the function

$$f(x) = \begin{cases} e^{x-1} + 2 & \text{for } x < 1 \\ tx^2 + 1 & \text{for } x \geq 1 \end{cases}.$$

Determine the real parameter t such that function f is continuous at $x_0 = 1$.

(b) Determine all convexity/concavity intervals of the function

$$f(x) = \frac{2 - e^x}{e^x + 2}.$$

(11 points)

3. Given is the function

$$C(x) = 4x^2 + 3 \qquad D_C = [0, \infty].$$

(a) Determine the rate of change $\varrho_C(x)$ and its local extreme points.

(b) Determine all $x \in D_C$ for which function C is elastic.

(11 points)

4. Using integration by substitution, find the integral

$$\int x^5 \cdot \sqrt{9 - x^3}\, dx\,.$$

(8 points)