

Working time: 60 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient. It is not allowed to use mobile phones or smart watches.

Tools:

- pocket calculator (according to the instructions of FWW)
- **either** one individually prepared one-sided A4 sheet of paper with arbitrary material (write '1' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet). If the formula sheet is used, please add your name and matriculation number and hand it in together with your examination.

Problems:

1. (a) Given is the alternating series $\{s_n\}$ with

$$s_n = \sum_{k=1}^n (-1)^k \cdot \frac{\sqrt{k}}{2k-1}.$$

Check by means of Leibniz's criterion whether the series $\{s_n\}$ converges.

- (b) Given is the function $f : \mathbb{R} \rightarrow \mathbb{R}$ with

$$f(x) = \begin{cases} e^{x-1} + 2 & \text{for } x < 1 \\ ax^2 + b & \text{for } x \geq 1 \end{cases}$$

Which relationship between the parameters a and b must hold so that function f is continuous at $x_0 = 1$?

(14 points)

2. Given is the function f with

$$f(x) = e^{-2x} \cdot \sqrt{x-1}.$$

Determine the domain D_f , all zeroes, monotonicity intervals, local extreme points and

$$\lim_{x \rightarrow \infty} f(x).$$

(15 points)

3. Given is the function f with

$$f(x) = e^{(x+1)^2}, \quad D_f = (0, \infty)$$

Determine all $x \in D_f$ for which function f is elastic.

(9 points)

4. (a) Find the integral

$$\int \sqrt{x} \cdot \ln x \, dx.$$

(b) Given are the functions

$$f(x) = x^2 + 2 \quad \text{and} \quad g(x) = \frac{2}{x} + 1$$

with $D_f = D_g = (0, \infty)$. Determine the composite function $f \circ g$, its range $R_{f \circ g}$ and the inverse function g^{-1} . Give **brief** arguments why the composition $f \circ g$ and the inverse function g^{-1} exist.

(11 points)