Klausur: 41050 Mathematical Methods I   Summer term 2018
Prüfer: apl. Prof. Dr. F. Werner

Working time: 60 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient. It is not allowed to use mobile phones or smart watches.

Tools:
- pocket calculator (according to the instructions of FWW)
- **either** one individually prepared one-sided A4 sheet of paper with arbitrary material (write '1' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet). The used material may not contain solved problems from tutorials, lectures and old exams. If the formula sheet is used, please add your name and matriculation number and hand it in together with your examination.

Problems:

1. (a) Determine all real solutions of the inequality

$$4 - |x - 2| < |2x + 1|.$$

   (b) Given is the series

$$\sum_{k=1}^{\infty} \frac{(k-1)!}{4^{2k+1}}.$$

   By means of the quotient criterion, check whether this series converges or not.
   (15 points)

2. Given are the functions
$$f(x) = 4x - 5, \quad D_f = (0, \infty)$$

   and

$$g(x) = \frac{2}{\sqrt{x}} + 3, \quad D_g = (0, \infty).$$

   (a) Determine the composite function $h(x) = (f \circ g)(x)$ and its range R_h.
   (b) Determine the inverse function $h^{-1}(x)$ and its domain $D_{h^{-1}}$ and range $R_{h^{-1}}$.
   (c) Does the composite function $k(x) = (g \circ f)(x)$ exist?
   (10 points)

3. Given is the function
$$f(x) = \frac{\ln(x-1)}{3(x-1)^2}.$$

   Determine the domain D_f, all zeroes and all local extreme points (including their type) of function f.
   (13 points)

4. (a) Given is the marginal cost function

$$C'(x) = \frac{4x}{\sqrt{3x^2 + 20}},$$

where the quantity x produced is given in hundreds of units and the marginal cost C' is given in thousands of EUR. Determine the cost function $C(x)$ and calculate the change in costs when production increases from 1000 units to 1500 units?

(b) Determine

$$L = \lim_{x \to 1} x^{\frac{2}{\sqrt{x}-1}}.$$

(**12 points**)