Klausur: 41053 Mathematical Methods II **Sommersemester 2016**
Prüfer: apl. Prof. Dr. F. Werner

Working time: 60 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator (according to the instructions of FWW)
- **either** one individually prepared one-sided A4 sheet of paper with arbitrary material (write '1' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet)

It is not allowed to use mobile phones.

Problems:

1. Given is the matrix

$$M = \begin{pmatrix} 2 & 4 & 2 \\ 3 & 1 & -7 \\ 2 & 2 & -2 \end{pmatrix}.$$

(a) Check whether matrix M is regular or singular.

(b) Determine matrix X satisfying the matrix equation

$$X \cdot N = M^2 + 2X\,,$$

where matrix M is given as above and

$$N = \begin{pmatrix} 2 & 1 & 0 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

(13 points)

2. Given are the matrix A and the vector $\mathbf{b}$ as follows:

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & u & 2 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} -4 \\ 2 \\ v \end{pmatrix},$$

(u, v are real parameters).

(a) Determine for which values of parameters u, v does the system $A\mathbf{x} = \mathbf{b}$ have

- a unique solution;
- no solution;
- infinitely many solutions.

(b) Give the general solution in the case of infinitely many solutions.

(9 points)

3. Given is the function $F : D_F \to \mathbb{R}$ with

$$F(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + x_3^3 + \frac{x_1}{x_3 - x_2} + x_2\sqrt{x_3}\ .$$

(a) Determine the gradient of function F at the point $\mathbf{x^0} = (x_1^0, x_2^0, x_3^0) = (8, 2, 4)$.

(b) Determine the directional derivative of function F at the point $\mathbf{x^0}$ in the direction given by vector $\mathbf{r} = (2, 1, 2)^T$.

(c) Determine the total differential dF of function F and use it to compute approximately the absolute and relative error in the computation of $F(\mathbf{x^0})$ when the independent variables are from the intervals $x_1 \in [7.8, 8.2], x_2 \in [1.9, 2.1], x_3 \in [3.9, 4.1]$.

(14 points)

4. (a) Determine all points satisfying the necessary conditions of the Lagrange multiplier method for a local extreme point of the function

$$f(x, y) = x^2 + y^2$$

subject to the constraint

$$x^2 + 2y^2 - 2 = 0\ .$$

(b) Using the sufficient conditions, check whether the point $(x^*, y^*; \lambda^*) = (-\sqrt{2}, 0; -1)$ is a local minimum or maximum point and give the corresponding function value.

(14 points)