Klausur: 41053 Mathematical Methods II Summer term 2017
Prüfer: apl. Prof. Dr. F. Werner

Working time: 60 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient. It is not allowed to use mobile phones or smart watches.

Tools:

- pocket calculator (according to the instructions of FWW)
- **either** one individually prepared one-sided A4 sheet of paper with arbitrary material (write '1' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet)

If the formula sheet is used, please add your name and matriculation number and hand it in together with your examination.

Problems:

1. Given are the vectors

$$\mathbf{u} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \qquad \text{and} \qquad \mathbf{v} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}.$$

   (a) Determine all vectors $\mathbf{x}$ which are orthogonal to vector $\mathbf{u}$ and which satisfy also $\mathbf{x}^T \cdot \mathbf{v} = 1$?

   (b) Among all solutions in (a), find the vector with

$$|\mathbf{x}| = \sqrt{\frac{19}{2}}\,.$$

   (c) Determine the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$.

   (13 points)

2. Given is the following system of linear equations:

$$\begin{array}{rcrcrcr} 2tx_1 & - & 9x_2 & - & tx_3 & = & 18 \\ 2x_1 & + & x_2 & + & x_3 & = & 0 \\ 2x_1 & + & x_2 & + & 2x_3 & = & 1 \end{array}$$

   (t is a real parameter).

   (a) By means of rank investigations characterize the cases when there exist infinitely many solutions, when there is a unique solution and when the system does not have a solution.

   (b) Let $t = -9$. Give the general solution for this case and the particular solution satisfying the condition that the sum of the variables x_1, x_2, x_3 is 5.

   (12 points)

3. Given is the function $f : D_f \to \mathbb{R}$ with

$$z = f(x, y) = 3\sqrt{2x + y} + 2x^2 y + \frac{\ln(x - 4)}{y} \, .$$

(a) Determine the gradient of function f at the point $(x_0, y_0) = (5, 6)$.

(b) It is known that the values of both variables are known only approximately: $x \in [4.98, 5.02]$ and $y \in [5.97, 6.03]$. Using the total differential, estimate the maximal absolute error in the function value for this case.

(c) Determine the value of the directional derivative in the direction $r = (1, 2)^T$ at the point $(x_0, y_0) = (5, 6)$.

(12 points)

4. Determine all local extreme points (including their type) of function

$$f(x, y) = 5 + \sqrt{2x + 7} + \sqrt{3y - \frac{3}{4}}$$

subject to the constraint

$$x + 3y = 4 \, .$$

(13 points)