Klausur: 41053 Mathematical Methods II **Summer term 2019**
Prüfer: apl. Prof. Dr. F. Werner

Working time: 60 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient. It is not allowed to use mobile phones or smart watches.

Tools:

- pocket calculator (according to the instructions of FWW)
- **either** one individually prepared one-sided A4 sheet of paper with arbitrary material (write '1' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet)

If the formula sheet is used, please add your name and matriculation number and hand it in together with your examination.

Problems:

1. Given is the following matrix B:

$$B = \begin{pmatrix} 2 & 4 & 2 \\ 3 & 1 & -7 \\ 2 & 2 & -2 \end{pmatrix}.$$

   (a) Check whether matrix B is regular or singular.

   (b) Determine matrix X satisfying the matrix equation

$$X \cdot A = B^2 + 2X\,,$$

   where matrix B is given as above and

$$A = \begin{pmatrix} 2 & 1 & 0 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

   (14 points)

2. Given is the following system of linear equations:

$$\begin{array}{rcrcrcrcl} x_1 & + & x_2 & + & x_3 & + & x_4 & = & 1 \\ 3x_1 & + & 2x_2 & + & x_3 & & & = & 0 \\ 2x_1 & + & x_2 & & & + & \lambda x_4 & = & \mu \end{array}$$

   (a) By means of rank investigations check for which values of $\lambda, \mu \in \mathbb{R}$ the given system is consistent/inconsistent and characterize the solution in dependence on λ and μ (give also the degrees of freedom).

(b) Consider the case $\lambda = 3$ and $\mu = 7$. Determine the general solution and the particular solution satisfying

$$x_1 + x_2 = 1 \; .$$

(13 points)

3. (a) Given is the function

$$z = f(x, y) = \frac{1}{xy} \; .$$

– Determine the total differential dz of function z and use it to compute approximately the change of $f(2, -3)$ when the independent variables change by $\Delta x = -0.1$ and $\Delta y = 0.2$.
– Determine the directional derivative at the point $(2, -3)$ into the direction given by the vector $(1, 3)^T$.
(b) The equation

$$F(x, y) = y^3 + xy - 12 = 0$$

defines an implicitly given function $y = f(x)$. For $y_0 = 1$, determine x_0 so that $F(x_0, y_0) = 0$ holds and determine $y'(x_0)$ by the implicit-function rule.

(11 points)

4. Consider the problem

$$f(x_1, x_2, x_3) = x_1 \; x_2 \; x_3 + 20 \to \text{max!}$$

s.t.

$$g(x_1, x_2, x_3) = x_1 + x_2 + x_3 = 9.$$

Determine all local maximum points by the Lagrange multiplier method. Take into account that a point, where at least one variable has the value 0, cannot be a local maximum point. Check also the sufficient condition and give the optimal function value.

(12 points)