Klausur: 41053 Mathematical Methods II **Winter term 2017/18**
Prüfer: apl. Prof. Dr. F. Werner

Working time: 60 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient. It is not allowed to use mobile phones or smart watches.

Tools:

- pocket calculator (according to the instructions of FWW)
- **either** one individually prepared one-sided A4 sheet of paper with arbitrary material (write '1' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet)

If the formula sheet is used, please add your name and matriculation number and hand it in together with your examination.

Problems:

1. Given are the vectors

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 1 \\ u \\ 1 \end{pmatrix} \qquad \text{and} \qquad \mathbf{c} = \begin{pmatrix} 0 \\ -2 \\ 3 \end{pmatrix}.$$

(u is a real parameter)

(a) Determine u such that vector $2\mathbf{a} + \mathbf{b}$ is orthogonal to vector $\mathbf{c}$.

(b) For which values of u do the vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ constitute a basis in $\mathbb{R}^3$.

(c) Determine the inverse matrix of the matrix A formed by the column vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ with $u = 2$.

(10 points)

2. Given is the following system of linear equations:

$$\begin{array}{rcrcrcrcl} x_1 & + & x_2 & + & x_3 & + & x_4 & = & 1 \\ 3x_1 & + & 2x_2 & + & x_3 & & & = & 0 \\ 2x_1 & + & x_2 & & & + & sx_4 & = & t \end{array}$$

(s, t real parameters).

By means of rank investigations discuss all possible cases for the solutions in dependence on the parameters s and t. In the case of infinitely many solutions, give also the degrees of freedom.

(9 points)

3. (a) The harvest H of an agricultural product depends on the amounts f_1, f_2, f_3 of the use of three fertilizers with $f_i \geq 0, i = 1, 2, 3$, as follows:

$$H(f_1, f_2, f_3) = -\frac{1}{3}(f_1)^3 + \frac{1}{2}(f_1)^2 - (f_2)^2 - \frac{1}{2}(f_3)^2 + f_1 f_2 + f_2 f_3 + 5f_1 + 3f_2 + 10\,.$$

Determine the amounts f_1, f_2, f_3 yielding a maximal harvest. Check also the sufficient conditions.

(b) The equation $F(x, y) = y - xy^2 + 2\cos x = 0$ defines an implicitly given function $y = f(x)$. Determine $y'(x = 0)$.

(17 points)

4. Given is the problem

$$F(x, y, z) = x + y + z + x(y + z) + yz + 100$$

subject to the constraint

$$2x + 3y + 4z = 191\,.$$

Does the point $(x_0, y_0, z_0) = (42, 25, 8)$ satisfy the necessary and sufficient condition for a local maximum point?

(14 points)