Klausur: 41053 Mathematical Methods II **Winter term 2018/19**
Prüfer: apl. Prof. Dr. F. Werner

Working time: 60 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient. It is not allowed to use mobile phones or smart watches.

Tools:

- pocket calculator (according to the instructions of FWW)
- **either** one individually prepared one-sided A4 sheet of paper with arbitrary material (write '1' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet)

If the formula sheet is used, please add your name and matriculation number and hand it in together with your examination.

Problems:

1. Given are the vectors

$$\mathbf{y} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \qquad \text{and} \qquad \mathbf{z} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}.$$

   (a) Determine all vectors $\mathbf{x}$ which are orthogonal to vector $\mathbf{y}$ and for which $\mathbf{x}^T \cdot \mathbf{z} = 1$.

   (b) Among the vectors found in (a), determine that vector for which $|\mathbf{x}| = \sqrt{29}$.

   (c) Determine the angle between the vectors $\mathbf{y}$ and $\mathbf{z}$.

   (12 points)

2. Given are the matrices

$$C = \begin{pmatrix} 9 & 5 & 3 \\ 3 & 5 & -5 \\ 12 & 7 & 8 \end{pmatrix} \qquad \text{and} \qquad D = \begin{pmatrix} 8 & 6 & 2 \\ 6 & 4 & -2 \\ 16 & 10 & 6 \end{pmatrix}.$$

   Determine matrix X satisfying the matrix equation

$$2X + X \cdot D = D + X \cdot C\,.$$

   (10 points)

3. The equation

$$F(x, y) = (x + 1)^2 + \ln[x(1 + y)] = 0$$

defines an implicitly given function $y = f(x)$. For $x_0 = -1$, determine y_0 so that $F(x_0, y_0) = 0$ holds. Determine $y'(x_0)$ at the resulting point (x_0, y_0) by the implicit-function rule.

(7 points)

4. (a) Given is the cost function

$$C(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + 3x_3^2 + x_1x_2 + x_2x_3 + 100\,,$$

where $x_i, i = 1, 2, 3$, denotes the produced amount of product i.

Currently, the produced amounts are $x_1 = 20, x_2 = 10, x_3 = 10$ (in tons). By means of the total differential, evaluate the relative error when the produced amounts are only known with a relative error of at most 10 %.

(b) The firm can increase the production by 5 tons in total and considers two variants: either to increase it according to the ratio $x_1 : x_2 : x_3 = 1 : 3 : 1$ or according to the ratio $x_1 : x_2 : x_3 = 2 : 1 : 2$. Using directional derivatives, determine the variant which leads to a lower increase in cost.

(c) Assume that the market prices for the goods are $p_1 = 40, p_2 = 50, p_3 = 80$. For the resulting profit function

$$P(x_1, x_2, x_3) = p_1x_1 + p_2x_2 + p_3x_3 - C(x_1, x_2, x_3)\,,$$

determine those values x_1, x_2, x_3 which maximize the profit. Check also the sufficient optimality condition.

(21 points)