EXERCISES CHAPTER 12

1. Consider the differential equation $y' = y$.

   (a) Draw the direction field.

   (b) Solve the differential equation by calculation. Find the particular solution satisfying $y(0) = 1$ resp. $y(0) = -1$.

   (c) Draw the particular solution from part (b) in the direction field of part (a).

2. Find the solutions of the following differential equations:

   (a) $y' = e^{x-y}$; (b) $(1 + e^x)yy' = e^x$ with $y(0) = 1$.

3. Let a curve go through the point $P : (1, 1)$. The slope (of the tangent line) of the function at any point of the curve should be proportional to the squared function value at this point. Find all curves which satisfy this condition.

4. The elasticity $\varepsilon_f(x)$ of a function $f : D_f \to \mathbb{R}$ is given by

$$\varepsilon_f(x) = 2x^2 \left(\ln x + \frac{1}{2} \right).$$

   Find function f as the general solution of a differential equation and determine a particular function f satisfying the equality $f(1) = 1$.

5. Let y be a function of t and $y'(t) = ay(T - t)/t$, where $0 < t \leq T$ and a is a positive constant. Find the solution of the differential equation for $a = 1/2$ and $T = 200$.

6. Check whether

$$y_1 = x, \qquad y_2 = x \ln x \qquad \text{and} \qquad y_3 = \frac{1}{x}$$

   form a fundamental system of the differential equation $x^3y''' + 2x^2y'' - xy' + y = 0$. Determine the general solution of the given differential equation.

7. Find the solutions of the following differential equations:

   (a) $y' - 2y = \sin x$; (b) $y'' - 2y' + y = 0$ with $y(0) = 1$ and $y'(0) = 2$;

   (c) $2y'' + y' - y = 2e^x$; (d) $y'' - 2y' + 10y = 10x^2 + 18x + 7.6$ with $y(0) = 0$ and $y'(0) = 1.2$.

8. Solve the following differential equations:

   (a) $y''' - 2y'' + 5y' = 2 \cos x$; (b) $y''' + 3y'' - 4y = 3e^x$.