

## EXERCISES CHAPTER 2

1. (a) Given is an arithmetic sequence  $\{a_n\}$  with a first term  $a_1 = 15$  and the difference  $d = 8$ . Find the term  $a_{101}$ .  
 (b) For an arithmetic sequence, the terms  $a_8 = 21$  and  $a_{10} = 25$  are known. Find difference  $d$  and the terms  $a_1$  and  $a_n$ .
2. Given is the sequence  $\{a_n\}$  with

$$a_n = 3 + \frac{n}{2}, \quad n \in \mathbb{N}.$$

- (a) Is sequence  $\{a_n\}$  decreasing or increasing?
  - (b) Is  $\{a_n\}$  a bounded sequence?
  - (c) Find a recursive formula for this sequence.
3. (a) Let  $\{a_n\}$  be a geometric sequence with the ratio of successive terms  $q = -2/3$  and the term  $a_7 = 64/243$ . Find the first term  $a_1$ . Which of the terms is the first with an absolute value less than 0.01?  
 (b) A geometric sequence has the terms  $a_2 = 6$  and  $a_7 = 2/81$ . Find the first term  $a_1$  and the ratio of successive terms  $q$ .
  4. Given are the sequences

$$\{a_n\} = \left\{ \frac{3}{n} - 5 \right\}, \quad \{b_n\} = \left\{ \frac{7 - 2n}{n^2} \right\} \quad \text{and} \quad \{c_n\} = \left\{ \frac{2^n}{n!} \right\}, \quad n \in \mathbb{N}.$$

Are these sequences monotone and bounded? Find the limits of the sequences if they exist.

5. Find the limits of the following sequences if they exist ( $n \in \mathbb{N}$ ).  
 (a)  $\{a_n\} = \left\{ \frac{2n^n}{(n+1)^n} \right\}$ ;      (b)  $\{b_n\} = \left\{ \frac{an^4 + n^3}{3n^3 + 4n} \right\}$ ,  $a \in \mathbb{R}$ ;  
 (c)  $\{c_n\}$  with  $c_n = c_{n-1}/2$ . Check it for  $c_1 = 1$  and  $c_1 = 4$ .
6. A textile company starts in its first year with 2,000 shirts and 1,000 trousers. The production of shirts increases by 500 shirts every year, and the number of trousers raises by 20 per cent every year.  
 (a) How many shirts and trousers are produced in the second, third and tenth year?  
 (b) What is the total output of shirts and trousers after 15 years (use given formulas)?
7. Find the limit  $s = \lim_{n \rightarrow \infty} s_n$  for the following series  $\{s_n\}$ :

$$(a) s_n = \sum_{k=1}^n \frac{(-2)^{k+1}}{5^k}; \quad (b) s_n = \sum_{k=1}^n \frac{1}{k(k+1)} \quad (\text{Hint: Find a formula for } s_n).$$

8. Check with respect to the real numbers  $x$  whether the following series  $\{s_n\}$  converge:

$$(a) \quad s_n = \sum_{k=1}^n \frac{x^k}{k}; \quad (b) \quad s_n = \sum_{k=1}^n \frac{k^2}{x^k}.$$

9. Consider the following series  $\{s_n\}$  with:

$$(a) \quad s_n = \sum_{k=1}^n (-1)^k \cdot \frac{4(k+2)}{k^2}; \quad (b) \quad s_n = \sum_{k=1}^n \frac{k^2}{k!};$$
$$(c) \quad s_n = \sum_{k=1}^n \left( \frac{k}{k+1} \right)^{k^2}; \quad (d) \quad s_n = \sum_{k=1}^n \left[ (-1)^k + \frac{1}{k} \right].$$

Find the first four terms of the series and calculate the partial sums  $s_1, s_2, s_3$  and  $s_4$ . Check whether the series converge.

10. Which is the best investment for a family wishing to invest 10,000 EUR for ten years:

- (a) an interest rate of 6 per cent annually for ten years;
- (b) an interest rate of 7 per cent for the first five years and 5 per cent for the next five years;
- (c) an interest rate of 7.5 per cent for the first two years, then 6 per cent for four years and 5.5 per cent for the last four years.

11. A grandfather wants to give his grandson 100,000 EUR at his 30th birthday. How much does he need to invest at the 18th birthday of the grandson if the bank offers a rate of interest of 6 per cent p.a.?

12. What interest rate will turn 2,000 EUR into 3,582 EUR after ten years?

13. Tom invests 10,000 EUR. The bank offers a rate of interest of 5 per cent p.a. What is the amount after ten years when interest is compounded annually, quarterly and monthly, respectively? Find the effective rate of interest  $i_{eff}$  for quarterly and monthly compounding.

14. Ann has given 5,000 EUR to a bank on 20 October 2003, and she wants to get the money back on 20 April 2006. What is the amount at the end if the bank has credited interest of 3 per cent p.a. compounded annually?

15. You decide to put away 200 EUR every month. Bank A offers simple interest of 6 per cent per annum, and bank B offers 0.48 per cent per month compounded monthly. After making 12 deposits, how much money do you have at the end of one year in banks A and B, respectively?

- (a) The deposits are made at the same time the interest is credited (i.e. at the end of each month).
- (b) The deposits are made at the beginning of each month and the interest is credited at the end.

16. A man wants to buy a car for 24,000 EUR. He decides to make a first installment of 25 per cent and the remainder to pay by a car loan with the following conditions: The borrowed sum will be paid back in one payment at the end of three years. During

these 36 months interest is to pay monthly at a per annum rate of 4.8 per cent. At the same time a sinking fund has to be set up in order to repay the loan. The man has to make monthly deposits into his sinking fund, which earns 3 per cent interest per annum compounded monthly. Find the sinking fund deposit. How much money does the man need each month for paying the sum of deposit and interest?

17. What is the annual deposit required to pay out 10,000 EUR after five years? Interest of 5 per cent per annum is compounded annually.
18. Find the present value of an annuity at a rate of 5 per cent p.a. and with a withdrawal of 6,000 EUR at the end of each of 15 years.  
How does the present value change if the withdrawal at the end of each month is 500 EUR, assuming an ordinary annuity at 5 per cent p.a.?
19. On 1 January 2015, Peter will become retired. He has made an installment of 20,000 EUR on 1 January 2000 and after it, he makes annual payments of 2,000 EUR every year up to 2014 on 1 January. The bank offers 5 per cent p.a. all over the time.
  - (a) What is the amount on 1 January 2015?
  - (b) Beginning with 31 January 2015, Peter wants to get a monthly withdrawal of 500 EUR. What is the present value of the annuity after ten years? Interest is compounded monthly at a rate of 5 per cent p.a.
20. A family has raised a mortgage of 150,000 EUR on their house. They can amortize the balance at 8.5 per cent p.a. for ten years.
  - (a) What are the annual payments? What is their total payment? Give a complete redemption table with the current amount of the loan at the beginning and at the end of each year, interest payment and amortization installment for all the ten years.
  - (b) Find the total payment if they repay 15,000 EUR plus the interest for the current amount of the loan every year (equal amortization installments).
  - (c) How many years does the family repay when the annual annuity is  $A = 18,000$  EUR?
21. A project which is going over two years has the expected costs and returns at the end of year  $k$  (in EUR) given in the following table:

Year	Costs	Returns
$k$	$C_k$	$R_k$
0	40,000	0
1	5,000	18,000
2	6,000	35,000

- (a) Let the rate of interest be  $i = 5$  per cent p.a. Check by means of the method of net present value and by the method of internal rate of return whether the project should be realized.
- (b) What are the percentage rates of interest (use only integers) for the project being advantageous?

- (c) In the third year, the costs for the project will be  $C_3 = 6,000$  EUR and the returns  $R_3 = 36,000$  EUR. Is it now profitable to realize the project with  $i = 5$  per cent p.a.?
22. A transport company has bought a bus for 100,000 EUR. After eight years, the bus is sold to another company for 44,000 EUR. Compare depreciation amounts and book values at the end of each of the eight years when the company uses
- (a) a linear depreciation,
  - (b) an arithmetic-degressive depreciation, or
  - (c) a geometric-degressive depreciation.