EXERCISES CHAPTER 3

1. Let $A = \{1, 2, 3, 4, 5\}$. Consider the relations

$$R = \{(1, 2), (1, 4), (3, 1), (3, 4), (3, 5), (5, 1), (5, 4)\} \subseteq A \times A$$

and

$$S = \{(a_1, a_2) \in A \times A \,|\, a_1 + a_2 = 6\}.$$

   (a) Illustrate R and S by graphs and as points in a rectangular coordinate system.

   (b) Which of the following propositions are true for relation R:

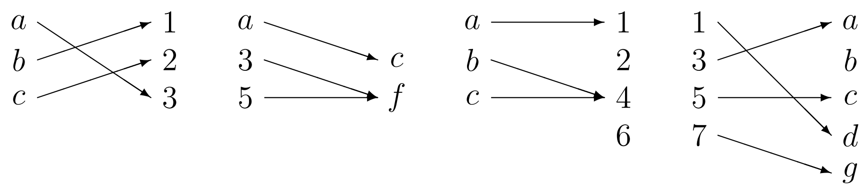
$$1R2, \quad 2R1, \quad 3\not R1, \quad \{2, 3, 4, 5\} = \{a \in A \mid 1Ra\}?$$

   (c) Find the inverse relations for R and S.

   (d) Check whether the relations R and S are mappings.

2. Consider the following relations and find out whether they are mappings. Which of the mappings are surjective, injective, bijective?

1. 2. 3. 4.



3. Let $A = B = \{1, 2, 3\}$ and $C = \{2, 3\}$. Consider the following mappings $f : A \to B$ and $g : C \to A$ with $f(1) = 3, f(2) = 2, f(3) = 1$ and $g(2) = 1, g(3) = 2$.

   (a) Illustrate $f, g, f \circ g$ and $g \circ f$ by graphs if possible.

   (b) Find the domains and the ranges of the given mappings and of the composite mappings.

   (c) What can you say about the properties of these mappings?

4. Given is the relation

$$F = \{(x_1, x_2) \in \mathbb{R}^2 \mid |x_2| = x_1 + 2\}.$$

   Check whether F or F^{-1} is a mapping. In the case when there exists a mapping, find the domain and the range. Graph F and F^{-1}.

5. Given are the relations

$$F = \{(x_1, x_2) \mid x_2 = x_1^3\} \quad \text{with } x_1 \in \{-3, -2, -1, 0, 1, 2, 3\}$$

and

$$G = \{(x, y) \in \mathbb{R}^2 \mid 9x^2 + 2y^2 = 18\}.$$

   Are these relations functions? If so, does the inverse function exist?

6. Given are the functions $f : D_f \to \mathbb{R}$ and $g : D_g \to \mathbb{R}$ with

$$f(x) = 2x + 1 \qquad \text{and} \qquad g(x) = x^2 - 2.$$

Find and graph the composite functions $g \circ f$ and $f \circ g$.

7. Given are the functions $f : \mathbb{R} \to \mathbb{R}_+$ and $g : \mathbb{R} \to \mathbb{R}$ with

$$f(x) = e^x \qquad \text{and} \qquad g(x) = -x.$$

   (a) Check whether the functions f and g are surjective, injective and bijective. Graph these functions.

   (b) Find f^{-1} and g^{-1} and graph them.

   (c) Find $f \circ g$ and $g \circ f$ and graph them.

8. Find $a \in \mathbb{R}$ such that $f : D_f = [a, \infty) \to \mathbb{R}$ with

$$y = f(x) = x^2 + 2x - 3$$

being a bijective function. Find and graph function f^{-1}.

9. Find domain, range and the inverse function for function $f : D_f \to \mathbb{R}$ with $y = f(x)$:

   (a) $y = \dfrac{\sqrt{x} - 4}{\sqrt{x} + 4}$; (b) $y = (x - 2)^3$.

10. Given are the polynomials $P_5 : \mathbb{R} \to \mathbb{R}$ and $P_2 : \mathbb{R} \to \mathbb{R}$ with

$$P_5(x) = 2x^5 - 6x^4 - 6x^3 + 22x^2 - 12x \qquad \text{and} \qquad P_2(x) = (x - 1)^2.$$

   (a) Calculate the quotient P_5/P_2 by polynomial division.

   (b) Find all the zeros of polynomial P_5 and factorize P_5.

   (c) Verify Vieta's formulae given in Theorem **??**.

   (d) Draw the graph of the function P_5.

11. Check by means of Horner's scheme whether $x_1 = 1, x_2 = -1, x_3 = 2, x_4 = -2$ are zeros of the polynomial $P_6 : \mathbb{R} \to \mathbb{R}$ with

$$P_6(x) = x^6 + 2x^5 - x^4 - x^3 + 2x^2 - x - 2.$$

Factorize polynomial P_6.

12. Find domain, range and the inverse function for each of the following functions $f_i : D_{f_i} \to \mathbb{R}$ with $y_i = f_i(x)$:

   (a) $y_1 = \sin x$, $y_2 = 2 \sin x$, $y_3 = \sin 2x$, $y_4 = \sin x + 2$ and $y_5 = \sin(x + 2)$;

   (b) $y_1 = e^x$, $y_2 = 2e^x$, $y_3 = e^{2x}$, $y_4 = e^x + 2$ and $y_5 = e^{x+2}$.

   Graph the functions given in (a) and (b) and check whether they are odd or even or whether they have none of these properties.

13. Given are the following functions $f : D_f \to \mathbb{R}$ with $y = f(x)$:

(a) $y = \ln x^4$; (b) $y = \ln x^3$; (c) $y = 3x^2 + 5$;

(d) $y = \sqrt{4 - x^2}$; (e) $y = 1 + e^{-x}$; (f) $y = \sqrt{|x| - x}$.

Find domain and range for each of the above functions and graph these functions. Check where the functions are increasing and whether they are bounded. Which of the functions are odd or even?