EXERCISES CHAPTER 5

1. Use the substitution rule for finding the following indefinite integrals:

(a) $\int e^{\sin x} \cos x \, dx$; (b) $\int \frac{\ln x}{x} \, dx$; (c) $\int \frac{5}{1-4x} \, dx$;

(d) $\int \frac{dx}{e^{3-2x}}$; (e) $\int \frac{x \, dx}{\sqrt{x^2+1}}$; (f) $\int \frac{x^3 \, dx}{\sqrt{1+x^2}}$;

(g) $\int \frac{e^{2x} - 2e^x}{e^{2x}+1} dx$; (h) $\int \frac{dx}{\sqrt{2-9x^2}}$; (i) $\int \frac{\cos^3 x}{\sin^2 x} dx$;

(j) $\int \frac{dx}{1-\cos x}$; (k) $\int \frac{dx}{2\sin x + \sin 2x}$.

2. Use integration by parts to find the following indefinite integrals:

(a) $\int x^2 e^x \, dx$; (b) $\int e^x \cos x \, dx$; (c) $\int \frac{x}{\cos^2 x} \, dx$;

(d) $\int \cos^2 x \, dx$; (e) $\int x^2 \ln x \, dx$; (f) $\int \ln(x^2+1) \, dx$.

3. Evaluate the following definite integrals:

(a) $\int_{-1}^{2} x^2 \, dx$; (b) $\int_{0}^{4} \frac{dx}{1+\sqrt{x}}$; (c) $\int_{0}^{\frac{\pi}{2}} \sin^3 x \, dx$;

(d) $\int_{0}^{4} \frac{x \, dx}{\sqrt{1+2x}}$; (e) $\int_{0}^{t} \frac{dx}{2x-1}, \quad t < \frac{1}{2}$; (f) $\int_{0}^{\frac{\pi}{2}} \sin x \cos^2 x \, dx$;

(g) $\int_{-1}^{0} \frac{dx}{x^2+2x+2}$.

4. A firm intends to precalculate the development of cost, sales and profit of a new product for the first five years after launching it. The calculations are based on the following assumptions:

- t denotes the time (in years) from the introduction of the product beginning with $t = 0$;
- $C(t) = 1,000 \cdot \left[4 - (2e^t)/(e^t+1)\right]$ is the cost as a function of $t \in \mathbb{R}, t \geq 0$;
- $S(t) = 10,000 \cdot t^2 \cdot e^{-t}$ are the sales as a function of $t \in \mathbb{R}, t \geq 0$.

(a) Calculate total cost, total sales and total profit for a period of four years.

(b) Find average sales per year and average cost per year for this period.

(c) Find the total profit as a function of the time t.

5. (a) Find

$$\int_0^{2\pi} \sin x \, dx$$

and compute the area enclosed by function $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = \sin x$ and the x-axis.

(b) Compute the area enclosed by the two functions $f_1 : \mathbb{R} \to \mathbb{R}$ and $f_2 : \mathbb{R} \to \mathbb{R}$ given by

$$f_1(x) = x^3 - 4x \qquad \text{and} \qquad f_2(x) = 3x + 6.$$

6. The throughput $q = q(t)$ (output per time unit) of a continuously working production plant is given by a function depending on time t:

$$q(t) = q_0 \cdot \left[1 - \left(\frac{t}{10}\right)^2\right].$$

The throughput decreases for $t = 0$ up to $t = 10$ from q_0 to 0. One overhaul during the time interval $[0, T]$ with $T < 10$ effects that the throughput goes up to q_0. After that it decreases like before.

(a) Graph the function q with regard to the overhaul.

(b) Let $t_0 = 4$ be the time of overhaul. Find the total output for the time interval $[0, T]$ with $T > 4$.

(c) Determine the time t_0 of overhaul which maximizes the total output in the interval $[0, T]$.

7. Determine the following definite integral numerically:

$$\int_0^1 \frac{dx}{1 + x^2}.$$

(a) Use approximation by trapeziums with $n = 10$.

(b) Use Kepler's formula.

(c) Use Simpson's formula with $n = 10$.

Compare the results of (a), (b) and (c) with the exact value.

8. Evaluate the following improper integrals:

(a) $\displaystyle\int_{-\infty}^{0} e^x \, dx$; (b) $\displaystyle\int_{1}^{\infty} \frac{dx}{x^2 + 2x + 1}$; (c) $\displaystyle\int_{0}^{\infty} \lambda e^{-\lambda x} \, dx$;

(d) $\displaystyle\int_{0}^{\infty} \lambda x^2 e^{-\lambda x} \, dx$; (e) $\displaystyle\int_{0}^{4} \frac{dx}{\sqrt{x}}$; (f) $\displaystyle\int_{0}^{6} \frac{2x - 1}{(x + 1)(x - 2)} \, dx$.

9. Let function f with
$$f(t) = 20t + 200$$
(in EUR) describe the annual rate of an income flow at time t continuously received over the years from time $t = 0$ to time $t = 6$. Interest is compounded continuously at a rate of 4 per cent p.a. Evaluate the present value at time zero.

10. Given are a demand function D and a supply function S depending on price p as follows:
$$D(p) = 12 - 2p \qquad \text{and} \qquad S(p) = \frac{8}{7}\,p - \frac{4}{7}\,.$$
Find the equilibrium price p^* and evaluate customer surplus CS and producer surplus PS. Illustrate the result graphically.