EXERCISES CHAPTER 9

1. Solve the following problems graphically:

(a)
$$z = x_1 + x_2 \to \min!$$
$$\begin{array}{rrrrrrr} \text{s.t.} & x_1 & + & x_2 & \geq & 2 \\ & x_1 & - & x_2 & \geq & 3 \\ & x_1 & + & 2x_2 & \leq & 6 \\ & x_1 & + & 4x_2 & \geq & 0 \\ & & & x_1, x_2 & \geq & 0 \end{array}$$

(b)
$$z = -x_1 + 4x_2 \to \min!$$
$$\begin{array}{rrrrrrr} \text{s.t.} & x_1 & - & 2x_2 & \leq & 4 \\ & -x_1 & + & 2x_2 & \leq & 4 \\ & x_1 & + & 2x_2 & \leq & 8 \\ & & & x_1, x_2 & \geq & 0 \end{array}$$

(c)
$$z = -x_1 - 2x_2 \to \min!$$
$$\begin{array}{rrrrrrr} \text{s.t.} & -x_1 & + & x_2 & \leq & 4 \\ & x_1 & + & 2x_2 & \leq & 11 \\ & 2x_1 & + & x_2 & \leq & 10 \\ & x_1 & & & \leq & 4 \\ & & & x_1, x_2 & \geq & 0 \end{array}$$

(d)
$$z = x_1 + x_2 \to \max!$$
$$\begin{array}{rrrrrrr} \text{s.t.} & x_1 & - & x_2 & \geq & 0 \\ & -x_1 & - & 2x_2 & \leq & 4 \\ & & & x_1, x_2 & \geq & 0 \end{array}$$

2. A craftsman has a free capacity of 200 working hours which he wants to use for manufacturing two products A and B. Production of one piece uses up 1 hour for A and 4 hours for B. The number of pieces of product A is at most 100. The number of pieces of product B must be at least 30, but more than three times the amount of A is impossible. Finally, products A and B can be sold at prices of 20 EUR and 27 EUR but the variable costs incurred amount to 10 EUR and 21 EUR per piece of products A and B, respectively?
What output combination should the craftsman choose in order to maximize the total profit? Find the solution of the problem graphically.

3. Find the standard forms of the following linear programming problems:

(a)
$$z = x_1 - 2x_2 + x_3 \to \min!$$
$$\begin{array}{rrrrrrrr} \text{s.t.} & x_1 & + & x_2 & + & x_3 & \leq & 7 \\ & 3x_1 & - & x_2 & + & x_3 & \geq & -4 \\ & & & & & x_1, x_2, x_3 & \geq & 0 \end{array}$$

(b)
$$z = -x_1 + 2x_2 - 3x_3 + x_4 \to \min!$$
$$\begin{array}{rrrrrrrrr} \text{s.t.} & 2x_1 & + & 2x_2 & - & x_3 & + & 3x_4 & = & 8 \\ & x_1 & & & + & 2x_3 & + & x_4 & \leq & 10 \\ & -2x_1 & & & + & 2x_3 & - & 3x_4 & \geq & 0 \end{array}$$
$$x_1 \leq 0, \quad x_2, x_3 \geq 0, \quad x_4 \in \mathbb{R}$$

4. (a) Find the optimal solution of the following problem graphically and by the simplex method:
$$z = x_1 + x_2 \to \max!$$
$$\begin{array}{rrrrrrr} \text{s.t.} & 3x_1 & + & 2x_2 & \leq & 6 \\ & x_1 & + & 4x_2 & \leq & 4 \\ & & & x_1, x_2 & \geq & 0 \end{array}$$

(b) Solve problem 9.1 (c) by the simplex method.

5. Solve the following problems by the simplex method:

(a)
$$z = 7x_1 + 4x_2 + 5x_3 + 6x_4 \to \max!$$
$$\begin{array}{rrrrrrrrrrr} \text{s.t.} & 20x_1 & + & 10x_2 & + & 12x_3 & + & 16x_4 & \leq & 400 \\ & & & & & x_3 & & & \leq & 5 \\ & x_1 & + & x_2 & + & x_3 & + & x_4 & \leq & 30 \end{array}$$
$$x_1, x_2, x_3, x_4 \geq 0$$

(b)
$$z = 2x_1 - 6x_2 \to \min!$$
$$\begin{array}{rrrrrrrrr} \text{s.t.} & 2x_1 & - & x_2 & & & \leq & 10 \\ & x_1 & - & 3x_2 & & & \leq & 15 \\ & 3x_1 & & & + & x_3 & = & 12 \\ & & & x_1, x_2, x_3 & & & \geq & 0 \end{array}$$

6. Solve problem 9.2 by the 2-phase simplex algorithm.

7. Solve the following linear programming problems by the simplex method:

(a)
$$z = x_1 + x_2 - x_3 \to \min!$$
$$\begin{array}{rrrrrrrrr} \text{s.t.} & 3x_1 & - & x_2 & - & 4x_3 & \leq & 0 \\ & x_1 & + & 2x_2 & & & \geq & 10 \\ & & & x_2 & + & 3x_3 & \geq & 4 \\ & & & x_1, x_2, x_3 & & & \geq & 0 \end{array}$$

(b)
$$z = x_1 + 2x_2 + x_3 \to \max!$$
$$\begin{array}{rrrrrrrrr} \text{s.t.} & x_1 & + & x_2 & & & \leq & 10 \\ & & & x_2 & + & x_3 & \leq & 14 \\ & x_1 & & & + & x_3 & \geq & 15 \\ & & & x_1, x_2, x_3 & & & \geq & 0 \end{array}$$

(c)
$$z = 2x_1 - x_2 + x_3 \to \max!$$
$$\begin{array}{rrrrrrrrrrr} \text{s.t.} & x_1 & + & x_2 & - & x_3 & - & x_4 & = & 4 \\ & 2x_1 & + & 3x_2 & - & x_3 & - & 2x_4 & = & 9 \\ & & & x_2 & + & 2x_3 & & & \geq & 3 \end{array}$$
$$x_1, x_2, x_3, x_4 \geq 0$$