

# INTAS Final Report

## **Title**

**Constraint Scheduling: Mathematical models, algorithms, and applications**

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Project co-ordinator: Peter Brucker

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## **Objective of the Project**

The objective of the project is to study classes of scheduling problems arising in connection with production planning and with the design of computer operating systems. Such problems are more complex than classical scheduling environments because in most cases several additional constraints have to be satisfied. In this project, models representing these problems will be formulated and basic solution methods will be investigated.

## **Research Activities**

### **Description of the research activities**

Most of the work done can be considered as basic research on scheduling problems. The objective was to formulate optimization models for solv-

ing complex scheduling problems in production planning and design of computer operating systems. Results have been obtained for batching problems, scheduling problems for hybrid or flexible shops, problems with operations of equal duration or with a fixed number of jobs.

Problems have been classified according to their complexity. For special situations, polynomial algorithms have been developed. For *NP*-hard problems, dynamic programming algorithms, polynomial approximation schemes, branch-and-bound methods, decomposition approaches, and heuristics such as local search methods have been applied. Also stability analysis of optimal or approximate solutions have been performed.

The research activities were carried out in accordance with those indicated in the Work Program of the INTAS co-operation agreement.

## Results

Research was undertaken in seven areas. The results obtained in each area are documented in this section. Workshop and conference activities are described. Also a list of papers, which have been prepared in the framework of the project and published or accepted for publication in different journals is given.

### Area 1: Batch scheduling

Batching or grouping of similar tasks is mainly done to avoid set-up times or set-up costs. It is widely used in flexible manufacturing. We refer to batch scheduling problems as those combining scheduling and batching decisions.

A review of the literature on the models that integrate scheduling with batching decisions is given in [8]. Details of the basic algorithms are given and other significant results are referenced. Special attention is paid to the design of efficient dynamic programming algorithms.

A dynamic programming algorithm for the problem of scheduling jobs of two part types in a shop consisting of three machines is presented in [2]. The processing of each job consists of two stages. The first stage is undertaken on the machine common to all jobs and the second stage is undertaken on the machine specific to a particular part type. Set-up times are necessary at the first stage to switch from processing a job of one part type to processing a job of another part type. Jobs of the same part type processed contiguously at the first stage form a batch. The objective is to find a batch schedule minimizing the makespan. An integer programming formulation and a heuristic algorithm was presented for this problem in the literature. Our algorithm provides an optimal solution in time polynomial in the number of jobs.

The problem of scheduling  $n$  jobs on a single machine in batches to minimize some regular criteria is studied in [3]. Jobs within each batch are processed

sequentially so that the processing time of a batch is equal to the sum of the processing times of the jobs contained in it. Jobs in the same batch are completed at the same time when the last job of the batch has finished its processing. A constant set-up time precedes the processing of each batch. The number of jobs in each batch is bounded by some value  $b$ . If  $b < n$ , then the problem is called bounded. Otherwise, it is unbounded. For both the bounded and unbounded problems, dynamic programming algorithms are presented for minimizing the maximum lateness, the number of late jobs, the total tardiness, the total weighted completion time, and the total weighted tardiness when all due dates are equal, which are polynomial if there is a fixed number of distinct job due dates or processing times. More efficient algorithms are derived for the unbounded problem of minimizing the maximum lateness and some special cases of both the bounded and unbounded problems in which all due dates and/or processing times are equal. Several special cases of the bounded problem are shown to be *NP*-hard. Thus, a comprehensive classification of the computational complexities of the special cases is provided.

The problem of scheduling groups of jobs on unrelated parallel machines in batches subject to group deadlines was considered in the literature. A classification of computational complexities of special cases was provided for the situation when all groups have equal deadlines. In [7] it is established that the problem is strongly *NP*-hard for the case of identical machines, different deadlines, unit processing times and unit set-up times, and equal numbers of jobs in each group. It is shown that a presence of machine deadlines instead of job deadlines does not make the problem easier.

The single-machine problem, which combines batch scheduling and a determination of a common due date, is studied in [6]. An important special case of this problem arises when there are equal set-up times and equal job processing times. Computational complexity of this case was indicated to be open, however. It is proved to be *NP*-hard.

A new model with a so-called batching machine is considered in [1]. A batching machine is a machine that can handle up to  $b$  jobs simultaneously. The problem of scheduling  $n$  jobs on a batching machine to minimize regular scheduling criteria, that is criteria which are non-decreasing in the job completion times, is studied. The jobs that are processed together form a batch, and all jobs in a batch start and finish at the same time. The processing time of a batch is equal to the largest processing time of any job in the batch. Two variants are analyzed: the unbounded model, where  $b \geq n$ ; and the bounded model, where  $b < n$ . For the unbounded model, a characterization of a class of optimal schedules is given, which leads to a generic dynamic programming algorithm allowing the problems of minimizing various regular cost functions to be solved efficiently. The unbounded problems of minimizing the weighted number of tardy jobs and the total weighted tardiness are proved to be *NP*-hard. For the bounded model, an  $O(n^{b(b-1)})$  dynamic programming algorithm is derived for minimizing total completion time; and for the case

with  $m$  different processing times, a dynamic programming algorithm that requires  $O(b^2m^22^m)$  time is given. It is proved that due date-based criteria give rise to  $NP$ -hard problems. It is shown that an arbitrary regular cost function can be minimized in polynomial time for a fixed number of batches.

Different constructive and iterative heuristic algorithms have been proposed in [4] for the permutation flow shop problem with batch processing, where the processing time of a batch consisting of jobs of the same group is given by the maximum of the processing times of the operations contained in the batch and the maximum batch size is limited. As an objective function, both the minimization of makespan and the weighted sum of completion time has been considered. Structural properties based on the critical path have been applied to guide the search in the case of makespan minimization. The algorithms developed have been tested on problems with up to 120 jobs.

Multi-operation jobs are to be scheduling on a single machine are discussed in [5]. Each job comprises several operations that belong to different families. Changeovers of production from one family to another have associated set-up times. A job completes when all of its operations have been processed. This problem combines the batching aspect of the well-known family scheduling models with an assembly element (where the job's operations are assembled to produce the final product). Three classic optimality criteria: the maximum lateness, the weighted number of late jobs, and the sum of job completion times are analyzed. The problem of minimizing the maximum lateness is equivalent to its counterpart without assembly. Thus, complexity results and algorithmic solutions are deduced. The number of late jobs problem is binary  $NP$ -hard when there are two families, and unary  $NP$ -hard when there are an arbitrary number of families, even when all set-up times are identical. For a fixed number of families, there is a a dynamic programming algorithm for the weighted number of late jobs problem which requires pseudopolynomial running time. A similar algorithm solves the sum of completion times problem to optimality, under the additional assumption that processing times are agreeable.

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## Area 2: Problems with operations of equal duration

The no-wait scheduling of  $n$  jobs, where each job is a chain of unit processing time operations to be processed alternately on two machines, is considered in [4]. The objective is to minimize the mean flow time. An  $O(n^6)$ -time algorithm to produce an optimal schedule is proposed. It is also shown that if zero processing time operations are allowed, then the problem is  $NP$ -hard in the strong sense.

The problem of minimizing the weighted number of late jobs in a two-machine job-shop with unit time operations is considered in [5].  $NP$ -hardness of the problem is established and a pseudo-polynomial algorithm is proposed, which is based on solving optimally a related problem where a maximal set of early jobs is determined.

An  $O(1)$  algorithm for minimizing arbitrary non-decreasing symmetric objective functions for processing  $n$  jobs with unit-time operations in an open shop is suggested in [9].

In [3], the problem of scheduling a set of unit-time jobs on  $m$  uniform machines is studied. Some jobs may require a unit of an additional single resource during their execution. The resource is renewable but the total resource consumption is limited by the same value at each time instant. The objective is to find a feasible schedule minimizing the maximum job completion time. It is shown that an approach suggested in the literature to solve this problem is incorrect. An  $O(m \log m)$  algorithm is presented for

the problem with no machine idle times and a linear time algorithm for the problem with identical machines.

The complexity status of a unit time open-shop  $O \mid r_i, p_{ij} = 1 \mid \sum U_i$  problem with release dates and the minimal number of late jobs criterion is investigated in [6]. It is shown by reduction from 3-partition that this problem is unary  $NP$ -hard.

The complexity status of a three-machine unit-time job-shop  $J3 \mid p_{ij} = 1 \mid \sum C_i$  scheduling problem is investigated in [7]. It is shown by reduction from 3-partition that the problem under consideration is unary  $NP$ -hard.

A two-machine open-shop problem to minimize the weighted mean flow time is considered in [8]. Processing times are suggested to be equal to 0 or 1. For this problem an  $O(n \log n)$ -algorithm is proposed.

The proportionate flow shop problem with controllable processing times is investigated in [1]. In this problem, each of the  $n$  jobs is processed by  $m$  machines in the same order and all operations of a single job have equal processing times and may be compressed by the same amount. The processing times of different jobs may be different. The objective is to minimize a sum of the makespan of the schedule and a compression cost function which is non-decreasing with respect to the amount of compression. An  $O(n \log n)$  algorithm for the following two types of compression cost functions is developed: convex and concave.

The single-machine problem of scheduling unit-time jobs with controllable release dates is considered in [2]. The release dates of the jobs which can be compressed incurring additional costs are given. The objective is to minimize the makespan of the schedule together with a linear compression cost function. An  $O(n^3)$  algorithm to construct a trade-off curve is developed.

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### Area 3: Problems with the fixed number of jobs

A polynomial algorithm for the job-shop scheduling problem with two machines and a fixed number of jobs is developed in [3]. The objective is to minimize the sum of weighted completion times. It is shown that this problem is solvable in polynomial time.

In [1] we show that problems  $J2 \mid n = 3, pmtn \mid C_{\max}$  and  $J2 \mid n = 3, pmtn \mid \sum C_i$  are shown to be binary *NP*-hard. Thus, the remaining open question concerning the complexity status of preemptive job-shop scheduling problems with fixed numbers of jobs and machines is settled. These results are the first demonstration that the problem with preemption is binary *NP*-hard, while its non-preemptive counterpart is polynomially solvable. Furthermore, a pseudopolynomial algorithm for problems  $Jm \mid n = k, pmtn \mid C_{\max}$  and  $Jm \mid n = k, pmtn \mid \sum C_i$  is presented which shows that problems  $J2 \mid n = 3, pmtn \mid C_{\max}$  and  $J2 \mid n = 3, pmtn \mid \sum C_i$  cannot be unary *NP*-hard.

For the non-preemptive two-machine job-shop scheduling problem with a fixed number of jobs and any regular objective function polynomial algorithms have been developed in [2]. This answers previous open questions about complexity status of job-shop problems with non-preemptive operations. For more special objective functions, structural properties of optimal schedules were derived. These properties are useful for the development of branch-and-bound methods.

The mixed-shop problems with a fixed number of jobs are considered in [6, 7, 4]. In the mixed shop, some jobs are processed with fixed routes like in the job shop, while the routes of the others are not fixed as in the open shop. In [6] results developed for mixed-shop problems with an unlimited

number of operations per job are presented. In [7, 4], results on computational complexity of mixed-shop problems are surveyed. The main attention is devoted to establishing the boundary between polynomially solvable and *NP*-hard problems.

A complexity analysis for the mixed-shop problems with a fixed number of jobs is done in [4]. Problem  $X2|n_J = 2, pmtn|C_{\max}$  is considered and an  $O(n + r^3)$  algorithm is proposed. It is a preemptive two-machine mixed-shop problem with  $n_J = 2$  jobs processed like in the job shop and an unlimited number  $n_O$  of jobs processed like in the open shop. Here  $n = n_J + n_O$  is the overall number of jobs and  $r$  is the maximal number of operations for jobs which are processed like in the job shop. Binary *NP*-hardness of problem  $X3|n_J = 2, n_O = 1, pmtn|C_{\max}$  (preemptive three-machine mixed-shop problem with  $n_J = 2$  and  $n_O = 1$ ) is established, and a pseudopolynomial algorithm is proposed. A number of algorithms for non-preemptive mixed-shop problems are developed on the basis of algorithms for non-preemptive “pure” shop problems.

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## Area 4: Generalized shop scheduling problems

In [11] a generalized shop scheduling problem is considered, where arbitrary precedence constraints among the operations are given (general shop), sequence-dependent changeover times between the processing of operations are taken into consideration, multiprocessor requirements of operations can occur, and in each technological stage of a job several identical machines of the corresponding type may be available (hybrid or flexible shop). However, the number of used machines of each type is not a priori fixed. As an objective function an arbitrarily given monotonically non-decreasing function in the completion times of the operations and the numbers of used identical machines of each type is considered. Based on the mixed multigraph model, two heuristic decomposition procedures and several algorithms for constructing and improving a schedule are proposed. The developed algorithms have been tested on a set of problem instances with up to 20 jobs and up to 10 machine types.

A scheduling problem in which each job has to be prepared before it can be processed is considered in [9]. The preparation is performed by a batching machine; it can prepare at most  $c$  jobs in each run, which takes an amount of time that is independent of the number and identity of the jobs under preparation. A strong Lagrangian lower bound is presented by using a new concept of positional completion times. This bound can be computed in  $O(n \log n)$  time only, where  $n$  is the number of jobs. Two classes of  $O(n \log n)$  heuristics that transform an optimal schedule for the Lagrangian dual problem into a feasible schedule are presented. Any heuristic in one class has a performance guarantee of  $3/2$ . It is shown that even the most naive heuristic in this class has a compelling empirical performance.

The problem of scheduling jobs on parallel machines with set-up times is considered in [10]. The set up has to be performed by a single server. The objective is to minimize the schedule length (makespan) as well as the forced idle time. A pseudopolynomial algorithm for the case of two machines when all set-up times are equal to one is presented. It is shown that the more general problem with an arbitrary number of machines is unary  $NP$ -hard and some list scheduling heuristics are analyzed for this problem. The problem of minimizing the forced idle time is known to be unary  $NP$ -hard for the case of two machines and arbitrary set-up and processing times. Unary  $NP$ -hardness of this problem, even for the case of constant set-up times, is proved and some polynomially solvable cases are given.

Scheduling problems to minimize the makespan, provided that the set-up phase of each operation needs to be attended by a single server, common for all jobs, are considered in [6]. For the processing system consisting of two parallel dedicated machines it is proved that the problem of finding an optimal schedule is  $NP$ -hard in the strong sense even if all set-up times are equal or if all processing times are equal. For the case of  $m$  parallel dedicated machines, a simple greedy algorithm is shown to create a schedule with the

makespan that is at most twice the optimum value. For the two-machine case, an improved heuristic guarantees a tight worst-case ratio of  $3/2$ . Several polynomially solvable cases of the latter problem are described. The two-machine flow shop and the open-shop problems with a single server are also shown to be *NP*-hard in the strong sense. However, the two-machine flow-shop no-wait problem with a single server reduces to the Gilmore–Gomory traveling salesman problem and is therefore solvable in polynomial time.

The problem of processing  $n$  jobs in a two-machine non-preemptive open shop to minimize the makespan is considered in [18]. One of the machines is assumed to be non-bottleneck. It is shown that, unlike its flow shop counterpart, the problem is *NP*-hard in the ordinary sense. On the other hand, the problem is shown to be solvable by a dynamic programming algorithm that requires pseudopolynomial time. The latter algorithm can be converted into a fully polynomial approximation scheme that runs in  $O(n^2/\epsilon)$  time. An  $O(n \log n)$  approximation algorithm is also designed which finds a schedule with makespan at most  $5/4$  times the optimal value, and this bound is tight.

Two models of two-stage processing with no-wait in the process are studied in [8]. The first model is the two-machine flow shop, and the other is the assembly model. For both models the problem of minimizing the makespan, provided that the set-up and removal times are separated from the processing times, is considered. Each of these scheduling problems is reduced to the traveling salesman problem (TSP). It is shown that, in general, the assembly problem is *NP*-hard in the strong sense. On the other hand, the two-machine flow shop problem reduces to the Gilmore–Gomory TSP, and is solvable in polynomial time. The same holds for the assembly problem under some reasonable assumptions.

A problem of sequencing  $n$  jobs in a two-machine re-entrant shop with the objective of minimizing the maximum completion time is considered in [1]. The shop consists of two machines,  $M_1$  and  $M_2$ , and each job has the processing route  $(M_1, M_2, M_1)$ . An  $O(n \log n)$  time heuristic is presented which generates a schedule with length at most  $4/3$  times that of an optimal schedule, thereby improving the best previously available worst-case performance ratio of  $3/2$ .

Two “minimum” *NP*-hard job-shop scheduling problems to minimize the makespan are investigated in [2]. In one of the problems every job has to be processed on at most two out of three available machines. In the other problem there are two machines, and a job may visit one of the machines twice. For each problem, a class of heuristic schedules is defined in which certain subsets of operations are kept as blocks on the corresponding machines. It is shown that for each problem the value of the makespan of the best schedule in that class cannot be substantially less than  $3/2$  times the optimal value. Algorithms that guarantee a worst-case ratio of  $3/2$  are presented.

The two-stage  $m$ -machine scheduling models with a bottleneck machine are studied in [3, 4, 5]. Here the processing of each of  $n$  jobs involves only two

operations, one of which is to be processed on a given machine, while the other operation must be processed on the bottleneck machine, common for all jobs. In the case of the open shop, it is shown that the problem of minimizing the makespan is solvable in  $O(n)$  time if  $m = 3$ , see [3]. In the case of an arbitrary  $m$ , a heuristic algorithm is developed that delivers a worst-case performance ratio of  $5/4$ , see [4]. For the job-shop version of the problem, a heuristic algorithm described in [5] guarantees a worst-case ratio of  $3/2$ .

The problem of minimizing the schedule length of a two-machine open shop in which a job can not only be assigned any of the two possible routes, but also the processing times depend on the chosen route, is considered in [19]. This problem is known to be *NP*-hard. A simple approximation algorithm that guarantees a worst-case performance ratio of 2 is proposed. Some modifications to this algorithm that improve its performance and guarantee a worst-case performance ratio of  $3/2$  are given.

The open-shop scheduling problem to minimize the makespan, provided that one of the machines has to process the jobs according to a given sequence, is studied in [14]. It is shown that in the preemptive case the problem is polynomially solvable for an arbitrary number of machines. If preemption is not allowed, the problem is *NP*-hard in the strong sense if the number of machines is variable, and is *NP*-hard in the ordinary sense in the case of two machines. For the latter case a heuristic algorithm is suggested that runs in linear time and produces a schedule with a makespan that is at most  $5/4$  times the optimal value. It is also shown that the two-machine problem in the non-preemptive case is solvable in pseudopolynomial time by a dynamic programming algorithm, and that the algorithm can be converted into a fully polynomial approximation scheme.

An extension of the greedy approach to finding an approximate solution to an open-shop problem to minimize the makespan is considered in [16]. In general, the algorithm reduces either a relative or an absolute error of the traditional greedy approach, and in the case of three machines guarantees a worst-case ratio of  $3/2$ .

The two-machine open-shop problem to minimize the makespan, provided that the jobs are transported from one stage to the other, is studied in [13] and [17]. The first of these papers develops a linear time algorithm, provided that the transportation times are small, and presents a heuristic for job-independent transportation times. The case of arbitrary times is handled in [17], where an algorithm with a worst-case ratio of  $3/2$  is given.

In [20] the problem of scheduling  $n$  jobs in a pallet-constrained flow shop with the objective to minimize the makespan is considered. In such a flow shop environment, each job needs a pallet the entire time, from the start of its first operation until the completion of the last operation, and the number of pallets in the shop at any given time is limited by an integer  $K$  with  $1 \leq K < n$ . Primarily the two-machine flowshop is considered and specifically the impact of the number of pallets on the makespan is studied. While it is an *NP*-hard

problem to find the minimum number of pallets subject to an upper bound on the makespan, a worst-case bound on the minimum  $K$  that guarantees the least possible makespan is proved. Furthermore, the empirical performance of Johnson's algorithm is investigated, which solves the problem to optimality if  $K \geq n$ , and Gilmore-Gomory's algorithm, which solves the problem to optimality if  $K = 2$ , when they are both straightforwardly adapted to deal with the situation where there are only  $K$  pallets available at any time and where  $2 < K < n$ . Computational experiments with randomly generated instances reveal that for Johnson's algorithm, which produces the least possible makespan, the required number of pallets grows quite rapidly with the number of jobs. In contrast, Gilmore-Gomory's algorithm, which may fail to produce the least possible makespan, consistently requires only about four pallets to produce schedules with a makespan very close to optimal.

In [15] we study the special case of an  $m$  machine flow shop problem in which the processing time of each operation of job  $j$  is equal to  $p_j$  is studied; this variant of the flow shop problem is known as the proportionate flow shop problem. It is shown that for any number of machines and for any regular performance criterion one can restrict the search for an optimal schedule to permutation schedules. Moreover, the problem of minimizing total weighted completion time is solvable in  $O(n^2)$  time.

In [7] a generalization of the permutation flow shop problem of scheduling  $n$  jobs on  $m$  machine centers is considered which is more adequate to some production scheduling problems, e.g. in the copper industry. Each center consists of parallel identical machines, each job consists of ordered operations that have to be processed on machines from different machine centers, the sequence in which the jobs pass through the machine centers is identically given for all jobs. The problem is to find an optimal sequence (schedule) in which  $n$  jobs pass through each machine center, this sequence having to be the same for each center. In addition to the hybrid flow-shop problem, the following generalizations are also considered, which make the problem more adequate to practical use. The processing times of the operations on some machines may vary between a minimum and a maximum value depending on the use of a continuously-divisible resource (this makes the problem more relevant for scheduling a copper electrorefining process in the copper industry). A non-regular criterion is based on due dates which are not a priori given but can be fixed by a decision-maker. A due date assignment cost is included into the objective function. Some properties of non-regular criteria for such a problem have been studied.

A job-shop scheduling problem with the objective of minimizing the sum of job completion times under uncertain numerical data is modeled in terms of a mixed graph in [12]. In contrast to well-studied deterministic settings, it is assumed that only the structural input data (i.e. precedence and capacity constraints) are fixed while for the operation processing times only their lower and upper bounds are known before scheduling and the probability distribution functions of the random processing times are unknown. The

structural input data are uniquely defined by the technological routes of the jobs, e.g. for a flow or open shop fixing the structural input data simply means fixing the numbers of jobs and machines. A mixed graph formulation of such a problem and an approach for dealing with such ‘strict uncertainty’ based on a stability analysis of an optimal schedule are developed. Dominance and strong dominance relations on the set of schedules are introduced and investigated. These relations are used for characterizations of a solution and a minimal solution of the scheduling problem with uncertain processing times. The formula for calculating the stability radius of an optimal schedule is given. The obtained results are used in a procedure for constructing potentially optimal schedules for all perturbations of the processing times within the given lower and upper bounds. To exclude redundant schedules, a stability analysis was used on the basis of pairwise comparison of schedules.

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## **Area 5: Local search heuristics**

In [3] a new binary encoding scheme for a single-machine scheduling problem to minimize total weighted tardiness has been introduced and tested in connection with multi-start descent, simulated annealing, threshold accepting,

tabu search, and genetic algorithms on large sets of test problems. The computational results show that the binary representation is more robust than the usual “natural” permutation representation.

A new technique, called *dynasearch*, is proposed in [5] which uses dynamic programming to search for the best move in a rather large neighborhood. The application of local search to problems which are a combination of a sequencing problem and another optimization problem are considered in [1]. For one problem mentioned in [1] an efficient procedure to calculate the best neighbor of a given solution for the adjacent pair interchange neighborhood is given in [4]. This procedure reduces the complexity of finding the best neighbor from quadratic to linear time. Based on these results, a technique similar to dynasearch is used to calculate the best neighbor for a new neighborhood of exponential size in quadratic time.

In [2] a tabu-search algorithm is presented and computational results are reported for the multi-mode job-shop problem. The multi-mode job-shop problem is a version of the multi-processor-task job-shop problem in which a set of machines must be assigned to each operation. The operation used all the assigned machines simultaneously during its processing time which depends on the assigned machines.

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## Area 6: Single-machine preemptive scheduling problems with deadlines, release and due dates

A single-machine preemptive scheduling problem to minimize the weighted number of late jobs is considered in [2]. Given are  $n$  jobs and for each job we have a release date, a processing time and a due date. It is assumed that certain specified jobs have to be completed on time. The due dates for these jobs are considered as deadlines, while for the other jobs due dates may be violated. For the case of similarly ordered release and due dates (when there is no advantage to preemption) as well as for the case of oppositely ordered release and due dates, the initial problem is transformed into a reduced problem, where all jobs with deadlines are eliminated.  $O(n \log n)$  algorithms are proposed for some special cases of the considered problems, for example when processing times and weights are oppositely ordered.

The single-machine due date assignment and scheduling problems with  $n$  jobs, in which the due dates are to be obtained from the processing times by adding a positive slack  $q$ , are studied in [3]. A schedule is feasible if there are no tardy jobs and the job sequence respects given precedence constraints. The value of  $q$  is chosen so as to minimize a function  $\varphi(F, q)$  which is non-decreasing in each of its arguments, where  $F$  is a certain non-decreasing earliness penalty function. Once  $q$  is chosen or fixed, the corresponding scheduling problem is to find a feasible schedule with the minimum  $F$ -value of function  $F$ . In the case of arbitrary precedence constraints, the problems under consideration are shown to be  $NP$ -hard in the strong sense even when  $F$  is total earliness. If the precedence constraints are defined by a series-parallel graph, both scheduling and due date assignment problems are proved to be solvable in  $O(n^2 \log n)$  time, provided that  $F$  is either the sum of linear functions or the sum of exponential functions. The running time of the algorithms can be reduced to  $O(n \log n)$  if the jobs are independent.

The single-machine problem of scheduling  $n$  independent jobs subject to target starting times is discussed in [4]. Target starting times are essentially release times that may be violated at a certain cost. The objective is to minimize an objective function that is composed of total completion time and maximum promptness, which measures the observance of these target starting times. It is shown that in the case of a linear objective function the problem is solvable in  $O(n^4)$  time if preemption is allowed or if the total completion time outweighs maximum promptness.

Column generation has proved to be an effective technique for solving the linear programming relaxation of huge set covering or set partitioning problems, and column generation approaches have led to state-of-the-art so-called branch-and-price algorithms for various archetypical combinatorial optimization problems. Usually, if Lagrangean relaxation is embedded at all in a column generation approach, then the Lagrangean bound serves only as a tool to fathom nodes of the branch-and-price tree. In [1] it is shown that the Lagrangean bound can be exploited in more sophisticated and effective ways



for two purposes: to speed up convergence of the column generation algorithm and to speed up the pricing algorithm. Our vehicle to demonstrate the effectiveness of teaming up column generation with Lagrangean relaxation is an archetypical single-machine common due date scheduling problem. Our comprehensive computational study shows that the combined algorithm is by far superior to two existing purely column generation algorithms: it solves instances with up to 125 jobs to optimality, while the purely column generation algorithm can solve instances with up to only 60 jobs.

In [5] and [6], fully polynomial approximation schemes are derived for the problem of scheduling  $n$  jobs on a single machine to minimize total weighted earliness and tardiness and for the problem of scheduling  $n$  deteriorating jobs on a single machine to minimize makespan, respectively. A new technique is used to develop the schemes. The main feature of this technique is that it recursively computes lower and upper bounds on the value of partial optimal solutions. Therefore, the scheme does not require any prior knowledge of lower and upper bounds on the value of a complete optimal solution.

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## Area 7: Stability analysis

The usual assumption that the processing times of the operations are known in advance is the strictest one in deterministic scheduling theory and it essentially restricts its practical aspects. A stability analysis of an optimal schedule may help to extend the significance of scheduling theory for some production scheduling problems. The terms 'stability', 'sensitivity' or 'postoptimal analysis' are generally used for the phase of an algorithm at which a solution (or solutions) of an optimization problem has already been found, and additional calculations are performed in order to investigate how this solution depends on the problem data. The stability radius of an optimal schedule for a general shop scheduling problem denotes the largest quantity of independent variations of the processing times of the operations such that this schedule remains optimal.

The calculation of the stability radius of an optimal or an approximate schedule for job shop scheduling problems is considered in [6]. Computational results on the calculation of the stability radius for randomly generated job-shop scheduling problems are presented. The extreme values of the stability radius are considered in more detail.

The influence of errors and possible changes of the processing times on the property of a schedule to be optimal for problems  $J|prec|\sum C_i$  and  $J|prec|C_{max}$  is investigated in [4]. A branch-and-bound method for the calculation of the stability radius of optimal schedules is used. Extensive numerical experiments with randomly generated job-shop scheduling problems are performed and discussed. Due to the developed software, we have the possibility to compare the values of the stability radii, the numbers of optimal schedules and some other 'numbers' for two often used criteria. The main question is how large the stability radius is, on average, for randomly generated job-shop problems.

Formulas for the calculation of the stability radius, when the objective is to minimize mean or maximum flow time for a general shop scheduling problem, are given in [5]. The extreme values of the stability radius are of particular importance, and these are considered in more detail. Computational results on the calculation of the stability radius for randomly generated job-shop scheduling problems are discussed.

The general shop problem with mean flow time objective function is considered in [1]. Necessary and sufficient conditions for the stability radius of optimal schedule to be equal to zero or to infinity are obtained. The formula for calculating the exact value of the stability radius is proposed.

In [2] a scheduling problem is considered with the objective of minimizing the makespan under uncertain numerical input data. The processing time of an operation, the release time and due date of a job are usually either assumed to be fixed before the scheduling in a deterministic model or to take some known probability distribution functions in a stochastic model.

This paper deals with the scheduling problem in which the structural input data (i.e. precedence and capacity constraints) are fixed before the stage of scheduling while all that is known before scheduling about the processing time of an operation, the job release time and due date are their upper and lower bounds. After improving the mixed graph model, an approach is developed for dealing with the scheduling problem under uncertain numerical data based on a stability analysis of an optimal makespan schedule. In particular, the candidate set of the critical paths in a circuit-free digraph is investigated, a minimal set of the optimal schedules is characterized and an optimal and a heuristic algorithm are developed. Computational results for randomly generated as well as well-known test problems are discussed.

Paper [3] deals with the general shop scheduling problem with the objective of minimizing the makespan under uncertain scheduling environments. The processing time of an operation is usually assumed to take a known probability distribution function when dealing with uncertain scheduling environments. The scheduling environments are supposed to be so uncertain that all information available about the processing time of an operation is an upper and lower bound. An approach to deal with such a situation based on an improved stability analysis of an optimal makespan schedule is presented and this approach is demonstrated on an illustrative example of the job-shop scheduling problem.

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## **Workshops and conferences**

The “Third Workshop on Models and Algorithms for Planning and Scheduling Problems” was organized with the participation of C.Glass, C.Potts and V.Strusevich in Cambridge, United Kingdom, April 7-11, 1997. At this international workshop scientists from 13 countries discussed scheduling problems.

15 members of the different groups within the INTAS-Project participated in the workshop and presented new project results.

On April 8, 1998 a business meeting for the INTAS-Project was organized. Members of the project groups discussed research projects and organized visits for NIS participants.

For the 16th European Conference on Operational Research which took place in Brussels, July 12-15, 1998 three INTAS Sessions were organized by the co-ordinator.

## Summary

### **Constraint Scheduling: Mathematical models, algorithms, and applications**

Reference number: INTAS-93-257 ext

Project co-ordinator: Peter Brucker

Period: June 1, 1997 to May 31, 1998.

Many scheduling problems that arise in production planning systems contain parameters and conditions not present in classical scheduling environments. To solve such problems it is necessary to construct specific methods and algorithms. Also, a technique for their complexity analysis is to be developed. Finding relevant methods and techniques is the objective of the project.

For a number of generalized shop scheduling problems where such parameters as set-up times, parallel machines on different processing stages, different types of jobs, their complexity status has been established, heuristic and approximate algorithms with guaranteed ratio performance have been suggested. For some special cases polynomial time algorithms for solving them to optimality have been given. Also the stability of optimal and approximate solutions for shop scheduling problems have been investigated.

The complexity of batch scheduling problems was studied and approximation algorithms for their solution developed.

In many situations problems with equal duration of operations or jobs arise. In another typical situation we have a fixed number of jobs in the processing system. For a number of such problems their complexity status has been established and exact and approximate polynomial time algorithms have been proposed.

For most of the complex scheduling problems considered it is impossible to find exact solutions in reasonable time. Therefore local search heuristics were developed and tested for some of the problems.