Fakultät für Mathematik
Institut für Mathematische Optimierung
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Examination in Mathematics I

(2 February 2012)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- **either** two individually prepared one-sided sheets of paper (write '2' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet)

It is not allowed to use mobile phones.

Distribution of points obtainable for the problems:

problem	1	2	3	4	5	6	sum
points	6	8	8	12	7	9	50

Problems:

1. Let
$$f(x) = x^4 + 2x(x+4) - 8(x+1) \ .$$
(a) Determine all (real and complex) zeroes of function f.

(b) Let the domain of function f be $D_f = [-3, 3]$. Does f^{-1} exist?

(c) Consider the function g with
$$g(x) = \frac{x(x^2-2)}{f(x)} \ .$$
Determine
$$\lim_{x \to \sqrt{2}} g(x) \ .$$
Give an argument why function g is not continuous at $x = \sqrt{2}$ and specify the type of discontinuity.

2. (a) Check by means of the quotient criterion whether the series
$$\sum_{k=1}^{\infty} \frac{10k^2}{(k+2) \cdot 4^{k+1}}$$
converges.

(b) Given is the complex number
$$z = -\frac{3}{2} + \frac{3}{2}\, i \ .$$
Determine $w = z^5$ in trigonometric form and the real part of the number w.

3. (a) You have won a price in a quiz program, and they offer you the following three options:

(a1) You get 50 000 EUR now.
(a2) You get 10 000 EUR now and in addition 5 000 EUR at the *end* of each of the following 10 years.
(a3) You get 4 000 EUR at the beginning of every year for 20

years (first time *now*).

Which choice is the best for you if an interest rate of 4 % p.a. is assumed over the whole period (and assuming that you will live at least for 20 more years)?

(b) What would be the smallest number of years with a payment of 5 000 EUR at the end of each year such that variant (a2) is the best?

4. Given is the function $f : D_f \to \mathbb{R}$ with $D_f = (0, \infty)$ and

$$f(x) = \frac{6x}{1 + x^2} \, .$$

   (a) Determine all local extreme points of function f.

   (b) Determine all intervals where function f is convex.

   (c) Determine for which values $x \in D_f$ function f is elastic.

5. Given is the function $f : D_f \to \mathbb{R}$ with

$$f(x) = \frac{1}{\sqrt{1 + 3x}} \, .$$

   (a) Expand function f at $x_0 = 0$ into a Taylor polynomial $P_3(x)$ of degree 3 (do *not* give the Lagrangian remainder).

   (b) Use the polynomial P_3 obtained in (a) to compute $\frac{1}{\sqrt{1.3}}$ approximately with three decimal places.

6. (a) Find the integral

$$\int \sqrt{x} \, \ln x \, dx \, .$$

   (b) Evaluate the integral

$$\int_0^1 \sqrt{x \, \sqrt{x}} \, dx \, .$$

   (c) Determine the area enclosed by the line $f(x) = 0$ and the function $f(x) = e^{\frac{x-1}{3}}$ within the interval $(-\infty, 4]$.