Fakultät für Mathematik
Institut für Mathematische Optimierung
Prof. Dr. F. Werner

Examination in 'Mathematics I'

(13 February 2013)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- **either** two individually prepared one-sided sheets of paper (write '2' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet)

It is not allowed to use mobile phones.

Distribution of points obtainable for the problems:

problem	1	2	3	4	5	6	sum
points	9	5	9	7	12	8	50

Problems:

1. (a) Determine all real numbers x satisfying the inequality

$$\frac{6}{2x+4} \leq |x| \ .$$

(b) Given are the complex numbers

$$z_1 = \sqrt{2} \cdot e^{i \cdot \frac{\pi}{4}} \qquad \text{and} \qquad z_2 = 2i \ .$$

Determine

$$z = \frac{z_2}{z_1 + \overline{z_2}}$$

both in the form $z = a + bi$ and in trigonometric form.

2. (a) How many years does it take to double a principal if interest of 3 % p.a. is credited and compounding takes place twice per year?

(b) Paul has 10,000 EUR available and needs 15,000 EUR after five years. The bank offers an interest rate of 4 % p.a. for the first installment of 10,000 EUR compounded annually and additional periodic payments at the end of each half year for which interest of 3 % p.a. is credited compounded semiannually. What is the necessary periodic payment at the end of each period so that the initial installment and the periodic payments result in 15,000 EUR after five years.

3. Given are the polynomials

$$P_3(x) = x^3 - 13x - 12 \qquad \text{and} \qquad P_5(x) = x^5 - 5x^4 - 10x^3 + 1 \ .$$

(a) Write function $f = P_5/P_3$ as the sum of a polynomial and a proper rational function.

(b) Determine all zeroes of function P_3.

(c) Determine

$$\lim_{x \to 1} \frac{2 \ln x}{\sqrt{3x^2 - 2} - 1} \ .$$

4. A firm has the cost function

$$C(x) = ux + 15 \ ,$$

where x denotes the produced output in units and $u \in \mathbb{R}$ is a parameter. The demand x, which is equal to the output, depends on the price p as follows:

$$x = F(p) = 300 - 15p \ .$$

(a) Formulate the profit function $G(p)$ in terms of the price variable p.

(b) Let $u = 6$. Find the profit-maximizing price p^*.

(c) Now let $u \in (0, 20]$ be arbitrary. Determine the profit-maximizing price p^* in dependence on the parameter u.

5. Given is the function $f : D_f \to \mathbb{R}$ with

$$y = f(x) = \lg(15 - 2x - x^2) - 1,$$

where lg denotes the logarithm to base 10.

(a) Determine the domain D_f and all zeroes.

(b) Determine all local extreme points of function f.

(c) Determine the interval(s), where function f is strictly increasing.

(d) Determine the interval(s), where function f is concave.

6. (a) Find

$$\int \frac{2}{x(3 - 2\ln x)} \, dx \ .$$

(b) Evaluate

$$\int_0^1 \ln(1 + \sqrt{x}) \, dx \ .$$