

Fakultät für Mathematik  
Institut für Mathematische Optimierung  
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**Examination in Mathematics I**

(15 July 2011)

**Working time:** 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

**Tools:**

- pocket calculator
- printed collection of formulas
- **either** two individually prepared one-sided sheets of paper (write '2' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet)

It is not allowed to use mobile phones.

**Distribution of points obtainable for the problems:**

problem	1	2	3	4	5	6	sum
points	4	6	7	9	13	11	50

### Problems:

1. Given is the complex number

$$z = \frac{3i}{(1+i)(2-i)}.$$

Determine the real part  $\operatorname{Re} z$  and the imaginary part  $\operatorname{Im} z$ .

2. (a) On January 1, 2012, Tom will retire. At this time, he has 80,000 € in a bank account, and he wants to arrange monthly withdrawals at the end of each month for a period of 15 years. If the bank offers a rate of interest of 3 % compounded monthly, what is the monthly withdrawal rate  $r$  so that at the end of 2026, no money is left over in his account.

(b) What is the amount left in his account after 10 years (i.e., at the beginning of 2022)?

3. Given is the polynomial  $P : \mathbb{R} \rightarrow \mathbb{R}$  with

$$P(x) = (x^3 - x^2 - 10x - 8) \cdot (x^3 + 27).$$

Determine all real and complex zeroes of  $P$ . If complex zeroes occur, given them in the Cartesian form  $a + bi$ .

4. (a) Given are the functions

$$f : [0, 5] \rightarrow R_f \quad \text{with} \quad f(x) = 3x + 1$$

and

$$g : [-1, 1] \rightarrow R_g \quad \text{with} \quad g(x) = \sqrt{1 - x^3}$$

Determine the ranges  $R_f$  and  $R_g$  and the composite functions  $f \circ g$  and  $g \circ f$  provided that they exist.

- (b) Determine

$$\lim_{x \rightarrow 2+0} \frac{\ln(2x - 3)}{\sqrt{x^2 - 4}}.$$

5. Given is the function  $f : D_f \rightarrow R_f$  with

$$f(x) = \frac{2}{3}\sqrt{4-x^2} \cdot x^3 .$$

(a) Find the domain  $D_f$  and zeroes of function  $f$ , and check whether  $f$  is even or odd.

(b) Find monotonicity intervals for function  $f$ , and determine all local extreme points and values.

(c) Give the range  $R_f$ .

(d) Does the inverse function  $f^{-1}$  exist (give an argument).

6. (a) Find the following integral:

$$\int \frac{2\sqrt{x}}{\sqrt{x}+1} dx$$

(b) Evaluate

$$\int_1^{\infty} \frac{\ln x}{x^2} dx$$