

Fakultät für Mathematik
Institut für Mathematische Optimierung
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Examination in Mathematics I
(20 July 2012)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- **either** two individually prepared one-sided sheets of paper (write '2' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet)

It is not allowed to use mobile phones.

Distribution of points obtainable for the problems:

problem	1	2	3	4	5	6	sum
points	8	7	8	10	9	8	50

Problems:

1. (a) Determine all complex numbers $z = a + bi$ satisfying the inequality

$$|2 - z| > |1 - 2z|.$$

- (b) Given is the complex number

$$z = \frac{2}{1 - i}.$$

Determine z^{10} in Cartesian form $a + bi$.

2. (a) Given is the series

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{4n^2}{2^{n+1}(n+1)}.$$

Check by means of the quotient criterion whether the above series converges.

- (b) Tom agrees to pay 300 EUR at the end of each month over a period of four years to pay off a loan for a boat. If interest of 6 % is charged monthly, how much did the boat originally cost and how much is the total interest payment?

3. Given is the function $f : \mathbb{R} \rightarrow \mathbb{R}_f$ with

$$f(x) = \sin 2x - \sin x.$$

- (a) Determine all zeroes of function f .
(b) Check whether function f is periodic.
(c) Check whether function f is even/odd.
(d) Now let the domain of function f be restricted to $D_f = [0, \frac{\pi}{2}]$. Does the inverse function f^{-1} exist?
4. (a) Given is the function $f : \mathbb{R} \rightarrow \mathbb{R}$ with

$$f(x) = \begin{cases} e^{x-1} + 2 & \text{for } x < 1 \\ ax^2 + b & \text{for } x \geq 1 \end{cases}$$

Determine a and b such that function f is continuous and differentiable at all points $x \in D_f$.

(b) Given is the function $f : \mathbb{R} \rightarrow \mathbb{R}$ with

$$f(x) = \frac{2 - e^x}{e^x + 2} .$$

Determine all intervals in which function f is convex and concave, respectively.

5. Given is the function $h : \mathbb{R}_+ \rightarrow \mathbb{R}$ with

$$h(x) = 4x^2 + t \quad (t > 0).$$

(a) Determine the rate of change ϱ_h of function h .

(b) Determine all local extreme points of the rate of change ϱ_h .

(c) Determine for which values $x \in D_h$ function h is elastic.

6. (a) Find the integral

$$\int \frac{\sqrt{2x}}{5 - \sqrt{2x}} dx .$$

(b) Determine

$$\lim_{x \rightarrow 2\pi+0} \frac{\sin \frac{x}{2}}{\sqrt{x - 2\pi}} .$$