

Fakultät für Mathematik
Institut für Mathematische Optimierung
Prof. Dr. F. Werner

Examination in Mathematics I

(31 July 2013)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- **either** two individually prepared one-sided sheets of paper (write '2' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet)

It is not allowed to use mobile phones.

Distribution of points obtainable for the problems:

problem	1	2	3	4	5	6	sum
points	13	7	6	9	7	8	50

Problems:

1. (a) Determine all $x \in \mathbb{R}$ satisfying the inequality

$$4 - |x^2 + 3x - 4| \geq -x .$$

- (b) Given is the polynomial $P_6(x) : \mathbb{R} \rightarrow \mathbb{R}$ with

$$P_6(x) = x^6 - x^4 - 20x^2 .$$

Determine all real and complex zeroes of P_6 .

- (c) Let $x = z^*$ be that complex zero found in (b) having a positive imaginary part. Determine all third zeroes of z^* in Cartesian form $w = a + bi$.

2. (a) Given is a geometric sequence $\{a_n\}$ with

$$a_4 = q = \frac{1}{3} .$$

Find a_1, a_n and s_n .

- (b) By means of the quotient criterion check for which $x \in \mathbb{R}$ the series $\{s_n\}$ with

$$s_n = \sum_{k=1}^n \left(\frac{x+2}{5} \right)^k$$

converges.

3. (a) Claudia has to repay a loan of 40 000 EUR over five years. At the end of each year, she has to make an installment consisting of a constant repayment plus interest. Find the annual installments when the interest rate is 8 % p.a. and determine the sum of all payments.

- (b) If Claudia would have installed a sinking fund five years ago with monthly payments at the end of each month for getting 40 000 EUR at the end of five years, what would be the monthly annuity A when the interest rate is 6 % p.a. compounded monthly. Determine the sum of all payments in these five years.

4. Given is the function $f : D_f \rightarrow R_f$ with

$$f(x) = \sqrt{x-1} + \sqrt{x+1} .$$

- (a) Determine the largest possible domain D_f and the range R_f of function f .
- (b) Determine all monotonicity and convexity/concavity intervals of function f .
- (c) Determine the inverse function f^{-1} of function f .

5. Given is the function $f : \mathbb{R}_+ \setminus \{0\} \rightarrow \mathbb{R}$ with

$$f(x) = 2e^{\frac{1}{8}x^2-x} \cdot x .$$

Determine all $x \in D_f$ with $\varrho_f(x) < 1$, where $\varrho_f(x)$ denotes the rate of change of function f .

6. (a) Evaluate the integral

$$\int_{\frac{1}{3}}^3 \frac{(1 - \sqrt{3x})^4}{\sqrt{3x}} dx$$

(b) Determine all zeroes and poles of function $f : D_f \rightarrow \mathbb{R}$ with

$$f(x) = \frac{\cos \frac{x}{3}}{(e^x - \sqrt{e})(x+2)} .$$