Fakultät für Mathematik
Institut für Mathematische Optimierung
Prof. Dr. F. Werner

Examination in Mathematics II

(6 February 2013)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- **either** two individually prepared one-sided sheets of paper (write '2' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet)

It is not allowed to use mobile phones.

Distribution of points obtainable for the problems:

problem	1	2	3	4	5	6	sum
points	7	12	9	5	10	7	50

Problems:

1. Given are the vectors

$$\mathbf{a^1} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \qquad \mathbf{a^2} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \quad \text{and} \qquad \mathbf{a^3} = \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}.$$

(a) Do the vectors $\mathbf{a^1}$, $\mathbf{a^2}$ and $\mathbf{a^3}$ constitute a basis?

(b) Express the vector

$$\mathbf{b} = \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix}$$

as a linear combination of the vectors $\mathbf{a^1}$, $\mathbf{a^2}$ and $\mathbf{a^3}$.

(c) Give all basises of $\mathbb{R}^3$ using a subset of the above vectors $\mathbf{a^1}, \mathbf{a^2}, \mathbf{a^3}$ and $\mathbf{b}$.

2. Given is the following matrix M:

$$\text{and} \qquad B = \begin{pmatrix} 2 & 4 & 2 \\ 3 & 1 & -7 \\ 2 & 2 & -2 \end{pmatrix}.$$

(a) Check whether matrix B is regular or singular.

(b) Determine matrix X satisfying the matrix equation

$$X \cdot N = M^2 + 2X\,,$$

where matrix M is given as above and

$$N = \begin{pmatrix} 2 & 1 & 0 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

3. Given is the following linear programming problem

$$z = -4x_1 - 2x_2 \to \min!$$

$$\begin{array}{lrcrcr} \text{s.t.} & 2x_1 & + & x_2 & \leq & 8 \\ & -x_1 & & & \geq & -3 \\ \end{array}$$

$$x_1 \geq 0,\ x_2 \geq 0.$$

(a) Determine all optimal basic feasible solutions by the simplex algorithm.

(b) Describe the set of all optimal solutions analytically.

(c) Does there exist an optimal solution of the above problem which is also optimal for the objective function

$$x_1 + 3x_2 \to \max!$$

4. Given is the matrix

$$C = \begin{pmatrix} -2 & u & 0 & 0 \\ u & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & v \end{pmatrix}$$

where u, v are real parameters. Determine the conditions on the two parameters such that matrix C is negative definite.

5. (a) Determine all points satisfying the necessary conditions for a local extreme point of function

$$f(x, y) = x^2 + y^2$$

subject to the constraint

$$x^2 + 2y^2 = 2 \; .$$

(b) Using the sufficient conditions, check whether the point $(x^*, y^*) = (\sqrt{2}, 0)$ is a local minimum or maximum point.

6. (a) Find all functions having the rate of change

$$\varrho_f(x) = \frac{3x}{x^2 + 1} + \sqrt{x} \, .$$

(b) Determine the particular solution x_p with $x_p(0) = 3$.