Fakultät für Mathematik
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Examination in Mathematics II

(28 July 2011)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- **either** two individually prepared one-sided sheets of paper (write '2' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet)

It is not allowed to use mobile phones.

Distribution of points obtainable for the problems:

problem	1	2	3	4	5	6	sum
points	8	9	7	7	10	9	50

Problems:

1. Given are the vectors

$$\mathbf{a} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \text{ and } \qquad \mathbf{c} = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}.$$

(a) Determine all vectors $\mathbf{x}$, which are orthogonal to the vector $\mathbf{a} - 2\mathbf{b}$ and for which $\mathbf{x}^T \cdot \mathbf{c} = 2$.

(b) Determine all vectors $\mathbf{x}$ from (a) with $|\mathbf{x}| = \sqrt{35}$.

2. Determine X from the matrix equation

$$X(M - N) - 4(M + X) = XN - 5X.$$

Calculate X when

$$M = \begin{pmatrix} 4 & 3 & 1 \\ 3 & 2 & -1 \\ 8 & 5 & 3 \end{pmatrix} \quad \text{and} \quad N = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & -2 \\ 2 & 1 & 2 \end{pmatrix}.$$

3. Given are the following matrix A and the vector $\mathbf{b}$ with

$$A = \begin{pmatrix} 1 & 2 & -3 & 4 \\ 3 & 5 & -4 & 6 \\ 4 & 5 & 3 & -2 \\ 3 & 8 & -19 & 24 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 2 \\ 8 \\ s \\ 2 \end{pmatrix},$$

where $s \in \mathbb{R}$.

(a) Is the system $A\,\mathbf{x} = \mathbf{b}$ consistent for all $s \in \mathbb{R}$?

(b) What is the value of $\det A^T$?

(c) Show that

$$\frac{1}{2} \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix}$$

is an eigenvector of A associated with the eigenvalue $\lambda = 0$.

4. Determine an optimal solution of the linear programming problem

$$z = 2\,x_1 - 2\,x_2 \to \text{min!}$$

subject to

$$\begin{array}{rcrcr} x_1 & + & x_2 & \leq & 5 \\ -x_1 & + & x_2 & \leq & 2 \\ x_1 & + & 2x_2 & \geq & 2 \\ & & x_1, x_2 & \geq & 0 \end{array}$$

by the simplex algorithm. Is the optimal solution uniquely determined (give an argument)?

5. Consider the problem

$$f(x, y, z) = x\;y\;z \to \text{max!}$$

s.t.

$$g(x, y, z) = x + y + z = c,$$

where c is a positive real parameter. Determine all local maximum points by the Lagrange multiplier method. Take into account that a point, where at least one variable has the value 0, cannot be a local maximum point. Check also the sufficient condition.

6. Determine the solution of the initial value problem

$$y'' + 6y' + 9y = 3x^2 + 7$$

$$y(0) = 2, \qquad y'(0) = \frac{5}{9}.$$