Fakultät für Mathematik
Institut für Mathematische Optimierung
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Examination in Mathematics II

(2 August 2012)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- printed collection of formulas
- **either** two individually prepared one-sided sheets of paper (write '2' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet)

It is not allowed to use mobile phones.

Distribution of points obtainable for the problems:

problem	1	2	3	4	5	6	sum
points	8	8	8	9	9	8	50

Problems:

1. Given is the following system of linear equations:

$$\begin{array}{rcrcrcr} x_1 & + & 3x_2 & - & 2x_3 & = & -4 \\ x_1 & + & 2x_2 & + & ux_3 & = & -v \\ 2x_1 & + & 5x_2 & - & 3x_3 & = & -6 \end{array}$$

   (a) For which values $u, v \in \mathbb{R}$ is the given system inconsistent?

   (b) For which values $u, v \in \mathbb{R}$ is the solution uniquely determined?

   (c) For which values $u, v \in \mathbb{R}$ do infinitely many solutions exist?

   (d) Determine the general solution for the case of infinitely many solutions and specify one basic solution.

2. Given is the following linear programming problem

$$\begin{array}{lrcrcr} & \multicolumn{5}{l}{z = -3x_1 - x_2 \rightarrow \min!} \\ \text{s.t.} & 3x_1 & + & x_2 & \leq & 18 \\ & 2x_1 & + & 3x_2 & \geq & 6 \\ & x_1 & - & 2x_2 & \geq & -8 \\ & \multicolumn{5}{l}{x_1, x_2 \geq 0.} \end{array}$$

   (a) Determine a **first** basic feasible solution by the simplex algorithm.

   (b) Determine graphically all optimal solutions and describe this set analytically.

3. Given is the matrix

$$A = \begin{pmatrix} 4 & -2 & 2 \\ -3 & 9 & -6 \\ -2 & 4 & -1 \end{pmatrix}$$

   (a) Verify that $\lambda = 3$ is an eigenvalue of matrix A.

   (b) Determine a maximal set of linearly independent eigenvectors associated with $\lambda = 3$.

   (c) Find all other real eigenvalues of matrix A.

4. (a) A function $y = f(x)$ is implicitly given by

$$F(x, y) = (x + 1)^2 + \ln[x(y + 3)] = 0 \ .$$

Determine the first derivative $f'(-1)$ at the point $x = -1$.

(b) Determine the directional derivative of function

$$f(x, y, z) = x^3 e^y + 2\sqrt{xz} + \ln(1 + 2xy)$$

at the point $(x^0, y^0, z^0) = (1, 0, 4)$ in the direction given by vector $\mathbf{r} = (2, -1, 2)^T$.

5. Determine all local extreme points of the function

$$f(x, y) = \sqrt{2x + 7} + \sqrt{3y - \frac{3}{4}}$$

subject to the constraint

$$x + 3y = 4 \ .$$

Check also the sufficient conditions.

6. Find the solution of the differential equation

$$y'' - 6y' + 9y = 9x + 12$$

satisfying the initial conditions

$$y(0) = 6, \qquad y'(0) = 15.$$