Fakultät für Mathematik
Institut für Mathematische Optimierung
apl. Prof. Dr. F. Werner

Examination in 'Mathematical Economics'

(12 February 2018)

Working time: 60 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- one individually prepared double-sided sheet of paper with arbitrary material
- textbook 'Mathematics of Economics and Business'

It is not allowed to use mobile phones or smart watches.

Problems:

1. Given are the function

$$f(x, y) = x^2 e^y - 3y + x$$

and the point $(x_0, y_0) = (-1, 0)$.

(a) Determine the equation of the tangent line to the level curve $f(x, y) = 0$ at the point (x_0, y_0) in explicit form.

(b) Given are the directions $\mathbf{r^1} = (1, 3)^T$ and $\mathbf{r^2} = (2, 1)^T$. Which of these two directions leads to a larger decrease in

the function value of function f when moving from the point (x_0, y_0) into the corresponding direction?

(c) Give the quadratic approximation of function f around (x_0, y_0).

(13 points)

2. Consider the nonlinear optimization problem:

$$f(x, y) = (x-1)^2 + e^{y^2} + a^2 \to \min! \qquad (a \in \mathbb{R})$$

s.t.

$$x^2 + y^2 \leq 1 \; .$$

(a) Setup the Karush-Kuhn-Tucker (KKT) conditions and find all solutions (x^*, y^*) of the KKT conditions. Check whether a globally optimal solution was found.

(b) For which values $a \in \mathbb{R}$ is the optimal function value at least equal to 10?

(15 points)

3. Given is the economic model

$$\begin{aligned} \dot{K} &= \sqrt{2K} + C - 6 \\ \dot{C} &= \frac{1}{4}K^3 - C + 2 \end{aligned}$$

(a) Determine the nullclines.

(b) Graph the nullclines and the sectors resulting from them and insert the directions of motions for all sectors. Determine the equilibrium point (K^*, C^*) with $K^* > 0, C^* > 0$ graphically.

(10 points)

4. Consider the following control problem:

$$\max \int_0^3 (1 - tx - u^2)dt \to \max! \qquad \dot{x} = u; \quad x(0) = 1; \quad x(3) \text{ free}$$

Formulate the necessary optimality conditions and determine a solution $(x^*(t), u^*(t))$ of the necessary conditions.

(12 points)