Fakultät für Mathematik
Institut für Mathematische Optimierung
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Examination in 'Mathematical Economics'

(28 January 2019)

Working time: 60 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator (according to the instructions of FWW)
- one individually prepared two-sided A4 sheet of paper with arbitrary material EXCEPT solved exercises, numerical examples from the lecture and old examination problems;
- textbook 'Mathematics of Economics and Business'

It is not allowed to use mobile phones or smart watches.

Problems:

1. Determine the quadratic approximation of function $F : \mathbb{R}^2 \to \mathbb{R}$ with
$$F(x, y) = e^{x \cdot e^{3y}} + xy^3$$
at the point $(x^0, y^0) = (0, 0)$.
(10 points)

2. Consider the nonlinear programming problem:
$$F(x, y) = x^2 + y + 15 \to \max!$$

subject to

$$\begin{array}{rcrcl} x^2 & + & y^2 & \leq & 9 \\ x & \geq & 0, & y & \geq 0 \end{array}$$

(a) Find all solutions (x^*, y^*) of the Karush-Kuhn-Tucker conditions.

(b) Is the sufficient condition for a global extreme point satisfied?

(14 points)

3. (a) Consider the following system of differential equations:

$$\begin{aligned} \dot{x} &= x - 4y \\ \dot{y} &= 2x - 5y \end{aligned}$$

Find the general solution by the eigenvalue method and transform then the given system into a differential equation of second order in x (do not solve it again).

(b) Determine the general solution of the separable differential equation

$$\frac{dy}{dt} = -2(t + e^{2t})y^2$$

and the particular solution satisfying $y(0) = \frac{1}{3}$.

(16 points)

4. Consider the following control theory problem:

$$\min \int_0^3 (4x+3u^2)dt, \quad \dot{x} = u, \quad x(0) = 0, \quad x(3) = 9, \quad u \in \mathbb{R}_+,$$

where $x(t)$ is the stock at time t and $u(t)$ is the production per unit of time.

(a) Formulate the necessary optimality conditions.

(b) Determine the only possible solution $(x^*(t), u^*(t))$ of the necessary optimality conditions.

(10 points)