Fakultät für Mathematik
Institut für Mathematische Optimierung
apl. Prof. Dr. F. Werner

Examination in 'Mathematical Economics'

(4 February 2020)

Working time: 60 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator (according to the instructions of FWW)
- one individually prepared two-sided A4 sheet of paper with arbitrary material EXCEPT solved exercises, numerical examples from the lecture and old examination problems;
- textbook 'Mathematics of Economics and Business'

It is not allowed to use mobile phones or smart watches.

Problems:

1. Determine the quadratic approximation of function $F : \mathbb{R}^2 \to \mathbb{R}$ with

$$F(x, y) = 2\left(e^{x \cdot e^{3y}} + xy^3\right) + e$$

at the point $(x^0, y^0) = (0, 0)$.

(14 points)

2. Consider the nonlinear programming problem:

$$F(x, y) = 10 - 4\ln(x^2 + 2) - y^2 \to \max!$$

s.t.

$$\begin{aligned} x &\geq 1 \\ x^2 &\geq 2 - y \end{aligned}$$

Setup the Karush-Kuhn-Tucker (KKT) conditions and find all solutions (x^*, y^*) of the KKT conditions.

(16 points)

3. Determine the general solution of the first-order differential equation

$$\frac{dy}{dx} = -4x^3y^2 - e^{2x-2}y^2$$

and the particular solution satisfying $y(1) = \frac{1}{4}$.

(9 points)

4. Given is the economic model

$$\begin{aligned} \dot{K} &= \frac{3}{2}\sqrt{K} - 2 - C \\ \dot{C} &= 7C^2 - C^3 - \frac{C^2K}{2} \end{aligned}$$

Determine the nullclines and the equilibrium point (K^*, C^*) with $K^* > 0, C^* > 0$ by **computation**.

(11 points)