

Fakultät für Mathematik
Institut für Mathematische Optimierung
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Examination in
‘Mathematical Methods in Business and Economics’
(13 February 2014)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- **either** two individually prepared one-sided sheets of paper (write ‘2’ on cover sheet) **or** textbook ‘Mathematics of Economics and Business (write ‘B’ on cover sheet)

It is not allowed to use mobile phones.

Distribution of points obtainable for the problems:

problem	1	2	3	4	5	6	sum
points	5	11	10	7	9	8	50

Problems:

1. (a) Given is the series

$$\sum_{n=1}^{\infty} \frac{3^{n-1}}{2^n \cdot n} .$$

Check whether this series converges.

(b) How long does it take to turn an amount of 15000 EUR into 22000 EUR if interest of 4 % p.a. compounded annually is credited?

2. Given is the function $f : \mathbb{R} \rightarrow \mathbb{R}$ with

$$f(x) = e^{-x} - e^{-4x} .$$

Determine all zeroes, local minimum and maximum points and inflection points of function f .

3. Given is the function $f : \mathbb{R}_+ \rightarrow \mathbb{R}$ with

$$f(x) = \frac{35x}{1+x^2} .$$

(a) Determine the first and second derivatives as well as all local extreme points of function f . Check whether they are local minimum or maximum points.

(b) Check whether there exist intervals, for which function f is elastic and determine them if appropriate.

4. (a) Find the integral

$$\int e^{\sqrt{x-1}} dx .$$

(b) Determine

$$\lim_{x \rightarrow 1} \frac{\cos(\pi x) + 1}{x^2 - 2x + 1} .$$

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5. Given is the matrix equation

$$X - XV = U - X(U - V).$$

- (a) Determine matrix X .
- (b) Compute X when

$$U = \begin{pmatrix} 4 & 3 & 1 \\ 3 & 2 & -1 \\ 8 & 5 & 3 \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & -2 \\ 2 & 1 & 2 \end{pmatrix}.$$

6. Given is the following system of linear equations:

$$\begin{aligned} x + 2y + 3z &= 2 \\ x + uy + 3z &= u \\ x + 2y + uz &= 0 \end{aligned}$$

(u is a real parameter).

By means of rank investigations check for which values of $u \in \mathbb{R}$ the given system

- (a) is inconsistent;
- (b) has a unique solution;
- (c) has infinitely many solutions.

Give the general solution in case (c).