Fakultät für Mathematik
Institut für Mathematische Optimierung
Prof. Dr. F. Werner

Examination in
'Mathematical Methods in Business and Economics'
(16 February 2015)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator (according to the instructions of FWW)
- **either** two individually prepared one-sided sheets of paper (write '2' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet)

It is not allowed to use mobile phones.

Distribution of points obtainable for the problems:

problem	1	2	3	4	5	6	sum
points	6	6	13	6	9	10	50

Problems:

1. Determine all $x \in \mathbb{R}$ satisfying the inequality

$$\frac{|2x - 2|}{x + 3} < 1 .$$

2. Paul has made an installment of 15,000 EUR on 1 January 2006 and in addition he makes 10 annual payments of 4,000 EUR at the beginning of every year from 2007 up to 2016. The bank offers 2.5 % p.a. all over the time compounded annually.

(a) What is the (overall) amount on 1 January 2017?

(b) At the beginning of 2017, Paul makes an additional payment so that now 70,000 EUR are in his account. Beginning at the end of 2017, Paul wants to get a annual withdrawal of 7,000 EUR. How long does it take until the account is empty if now interest of 3 % p.a. is offered compounded annually (i.e., for how many years does the bank pay annually 7,000 EUR at the end of each year?)

3. Given is the function $f : D_f \to R_f$ with

$$f(x) = 8x^2 + \frac{1}{x} .$$

(a) Determine the domain D_f and all zeroes of function f.

(b) Determine all local extreme points. Check function f for monotonicity and convexity/concavity, and determine all inflection points.

(c) Determine the limit

$$\lim_{x \to 0} f(x)$$

and graph function f.

4. (a) Evaluate

$$I = \int_{\frac{1}{2}}^{8} \frac{(\sqrt{2x} + 3)^3}{\sqrt{2x}} \, dx .$$

(b) Determine the limit

$$\lim_{x \to 0+0} \frac{x \cos x}{\sin x} .$$

5. Given is the matrix equation

$$A \cdot X \cdot B = 4I ,$$

where I is a (3×3) identity matrix and A and B are as follows:

$$A = \begin{pmatrix} -\frac{5}{3} & \frac{13}{6} & -\frac{1}{2} \\ \frac{4}{3} & -\frac{5}{6} & \frac{1}{2} \\ \frac{2}{3} & -\frac{1}{6} & \frac{1}{2} \end{pmatrix} , \qquad B = \begin{pmatrix} 3 & 4 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix} .$$

Solve the equation for matrix X and compute it.

6. Given is the following system of linear equations:

$$\begin{aligned}
x_1 &- x_2 + 4x_3 &= 0 \\
2x_1 &- 4x_2 + 6x_3 &= 2 \\
&\ 3x_2 - x_3 &= 5 \\
2x_1 &- 2x_2 + ax_3 &= b
\end{aligned}$$

$(a, b$ are real parameters).

(a) Check for $a = 4$ and $b = 0$ whether the system is consistent (by means of the rank criterion).

(b) Give the solution for $a = 8$ and $b = 0$.

(c) Determine b as a function of a such that the system is always consistent.