Fakultät für Mathematik
Institut für Mathematische Optimierung
Prof. Dr. F. Werner

Examination in
'Mathematical Methods in Business and Economics'
(3 August 2013)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator
- **either** two individually prepared one-sided sheets of paper (write '2' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet)

It is not allowed to use mobile phones.

Distribution of points obtainable for the problems:

problem	1	2	3	4	5	6	sum
points	9	5	10	7	9	10	50

Problems:

1. (a) Given is the alternating series $\{s_n\}$ with

$$s_n = \sum_{k=1}^{n} (-1)^k \frac{\sqrt{k}}{2k-1} \; .$$

By means of Leibniz's criterion show that the series $\{s_n\}$ converges.

(b) A loan of 50000 EUR should be amortized at a rate of interest of 7 % in 6 years by equal payments at the end of each month. What is the monthly annuity?

2. Given is the function $f : (-\infty, 4] \to R_f$ with

$$f(x) = \begin{cases} 2x - 2 & \text{for } x \in (-\infty, 2] \\ a + (x-2)^2 & \text{for } x \in (2, 4] \end{cases}$$

(a) For which $a \in \mathbb{R}$ is function f continuous?
(b) Let $a = 4$. Determine the inverse function f^{-1} and its domain.

3. Given is the function $f : D_f \to R_f$ with

$$f(x) = \frac{3x^3}{2x^2 + 4x + 2} \; .$$

Determine all discontinuities and their type, all local extreme points and monotonicity intervals.

4. Given are the marginal cost C_1' and C_2' of two different products P_1 and P_2 as follows:

$$C_1'(x) = 5xe^{-x^2} \, , \qquad C_2'(t) = \frac{16t}{\sqrt{100 + 2t^2}} \; .$$

(a) Determine all cost functions $C_1(x) = \int C_1'(x)dx$ which are possible for product P_1.

(b) Evaluate the cost function of product P_2 for $0 \le t \le 5$.

5. Given are the vectors

$$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}, \qquad \text{and} \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}.$$

(a) Determine all vectors $\mathbf{x}$ which are orthogonal to vector $\mathbf{b}$ and which satisfy $\mathbf{x}^T \cdot \mathbf{a} = 3$.

(b) Among all vectors $\mathbf{x}$ found in (a) determine those vectors $\mathbf{x}^*$ with $|\mathbf{x}^*| = \sqrt{18}$.

(c) Determine the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$

6. Given is the following system of linear equations:

$$\begin{array}{rcccccccl} x_1 & + & x_2 & + & x_3 & + & x_4 & = & 1 \\ 2x_1 & + & x_2 & & & + & ax_4 & = & b \\ 3x_1 & + & 2x_2 & + & x_3 & & & = & 0 \end{array}$$

(a) By means of rank investigations check for which values of $a, b \in \mathbb{R}$ the given system is consistent/inconsistent and characterize the solution in dependence on a and b (give also the degrees of freedom).

(b) Consider the case $a = 3$ and $b = 7$. Determine the general solution and the particular solution satisfying

$$x_1 + x_2 = 1 \ .$$