

Fakultät für Mathematik  
Institut für Mathematische Optimierung  
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**Examination in**  
**‘Mathematical Methods in Business and Economics’**  
(21 July 2014)

**Working time:** 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

**Tools:**

- pocket calculator
- **either** two individually prepared one-sided sheets of paper (write ‘2’ on cover sheet) **or** textbook ‘Mathematics of Economics and Business (write ‘B’ on cover sheet)

It is not allowed to use mobile phones.

**Distribution of points obtainable for the problems:**

problem	1	2	3	4	5	6	sum
points	6	7	12	7	8	10	50

### Problems:

1. Lena becomes retired on 1 February 2020. She has made an installment of 25000 EUR on 1 January 2008 and in addition she makes annual payments of 3000 EUR at the beginning of every year from 2009 up to 2020 (included). The bank offers 3 % p.a. all over the time compounded annually.
  - (a) What is the amount on 1 January 2020?
  - (b) Beginning with 1 February 2020, Lena wants to get a monthly withdrawal of 400 EUR. What is the present value of the annuity after eight years if again interest of 3 % is paid compounded monthly?
2. Given are the functions  $f : \mathbb{R} \setminus \{3\} \rightarrow R_f$  and  $g : \mathbb{R} \setminus \{0\} \rightarrow R_g$  with

$$f(x) = 1 - \frac{x - 5}{x - 3} \quad \text{and} \quad g(x) = \frac{5 - x}{x} .$$

- (a) Determine the ranges  $R_f$  and  $R_g$ .
- (b) Determine the composite functions

$$f \circ g \quad \text{and} \quad g \circ f$$

provided that they exist.

3. Given is the function  $f : D_f \rightarrow R_f$  with

$$f(x) = \sqrt{4x - 4} \cdot e^{-2x} .$$

- (a) Determine the domain, all zeroes, all local extreme points and all monotonicity intervals of function  $f$ .
- (b) Determine

$$\lim_{x \rightarrow \infty} f(x).$$

4. Given is the following function  $f$ :

$$f(x) = e^{(x+1)^2}, \quad D_f = (0, \infty)$$

- (a) Determine the elasticity  $\epsilon_f(x)$  for this function.
- (b) Check whether function  $f$  is elastic at  $x_0 = 1$ .
- (c) Determine all  $x \in D_f$  for which  $f$  is elastic.

5. (a) Determine

$$I = \int x^5 \cdot \sqrt{6 - x^3} dx .$$

(b) Determine all solutions of the equation

$$\log_3 [4 - \log_2 (x + 1)] = 1 .$$

6. Given are the matrix  $A$  and the vector  $\mathbf{b}$  as follows:

$$A = \begin{pmatrix} 1 & 0 & -1 \\ -1 & u & 3 \\ 3 & 1 & v \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

( $u, v$  are real parameters).

(a) Determine the rank of matrix  $A$  in dependence on the parameters  $u$  and  $v$ .

(b) Let  $u = 1$ . For which values of  $v$  are the column vectors of matrix  $A$  linearly independent.

(c) Now let  $u = 2$  and  $v = -2$ . Give the general solution of the system of equations  $A\mathbf{x} = \mathbf{b}$  for this case.