Fakultät für Mathematik
Institut für Mathematische Optimierung
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Examination in
'Mathematical Methods in Business and Economics'
(27 July 2015)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator (according to the instructions of FWW)
- **either** two individually prepared one-sided sheets of paper (write '2' on cover sheet) **or** textbook 'Mathematics of Economics and Business (write 'B' on cover sheet)

It is not allowed to use mobile phones.

Distribution of points obtainable for the problems:

problem	1	2	3	4	5	6	sum
points	9	7	13	5	8	8	50

Problems:

1. (a) Given is the series

$$\sum_{n=1}^{\infty} \frac{(n+1)^2}{2n \cdot 4^n} \ .$$

Check by means of the quotient criterion whether the above series converges.

(b) A firm produced 8,000 laptops in its first year 2011. How many laptops did the firm produce altogether up to the end of 18 years when the production increases by 15 % each year? What is the first year with a production of more than 60,000 laptops?

2. Given are the functions $f : (0, \infty) \to R_f$ and $g : (0, \infty) \to R_g$ with

$$f(x) = 3x - 2 \qquad \text{and} \qquad g(x) = \frac{1}{x} + 2$$

(a) Determine the composite functions $h = f \circ g$ and $k = g \circ f$ provided that they exist.

(b) Determine the inverse function h^{-1} if it exists.

3. Given is the function $f : D_f \to R_f$ with

$$f(x) = \frac{3x - 3}{2x^3} \ .$$

(a) Determine the domain D_f and all zeroes of function f.

(b) Calculate

$$\lim_{x \to \pm\infty} f(x) \qquad \text{and} \qquad \lim_{x \to 0} f(x) \ .$$

(c) Determine all local extreme points and inflection points. Give all monotonicity (i.e., where function f is increasing and decreasing, respectively) and concavity/convexity intervals.

4. Find the integral

$$\int \frac{\sqrt{x}}{\sqrt{x} + 2} \, dx \ .$$

5. Given are the vectors **a** and vector **b** as follows:

$$\mathbf{a} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}.$$

(a) Determine all vectors **x** for which

$$\mathbf{x} \perp \mathbf{a} \quad \text{and} \quad \mathbf{x} \cdot \mathbf{b} = -10.$$

(b) Among the vectors of the form

$$\mathbf{x} = \begin{pmatrix} 4 + t \\ -2 - 2t \\ t \end{pmatrix}$$

with $t \in \mathbb{R}$ find all vectors which satisfy

$$|\mathbf{x}| = \sqrt{26}.$$

6. Given are the matrix A and vector **b** as follows:

$$A = \begin{pmatrix} 1 & 2 & 4 & -3 \\ 3 & 5 & 6 & -4 \\ 4 & 5 & -2 & 3 \\ 3 & 8 & 24 & -19 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 2 \\ 8 \\ u \\ 2 \end{pmatrix},$$

where u is a real parameter.

(a) Is the system $Ax = b$ consistent for all $u \in \mathbb{R}$?

(b) Determine the general solution for $u = 14$.

(c) Give $\det A$.