Fakultät für Mathematik
Institut für Mathematische Optimierung
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Examination in 'Methods for Economists'

(10 February 2015)

Working time: 120 minutes

The derivation of the results must be given clearly. The statement of the result only is not sufficient.

Tools:

- pocket calculator (according to the instructions of FWW)
- two individually prepared sheets of paper with arbitrary material except solved exercises, numerical examples from the lecture and old examination problems
- textbook 'Mathematics of Economics and Business'

It is not allowed to use mobile phones.

Distribution of points obtainable for the problems:

problem	1	2	3	4	5	sum
points	13	12	8	10	7	50

Problems:

1. Given is the optimization problem

$$f(x, y) = x^2 + y^2 \to \text{min! (or max!)}$$

s.t.

$$g(x, y) = x^2 + xy + y^2 = a,$$

where a is a non-negative real parameter.

(a) Determine all points satisfying the necessary conditions for a local extreme point.

(b) Among the points found in (a), check the sufficient condition for a local extreme point for the point having the largest x-value and determine its type. Give its function value.

2. Given is the nonlinear programming problem

$$F(x, y) = x^3 - 2x - y \to \text{min!}$$

s.t.

$$x + y \leq s$$
$$x \geq 0, \; y \geq 0$$

($s \in [1, 2) \subset \mathbb{R}$).

(a) Determine all solutions of the KKT-conditions.

(b) Is the solution obtained globally optimal (give an argument).

(c) What is the condition on parameter s such that the optimal function value is at most equal to -2.

3. Consider the following system of differential equations:

$$\begin{aligned} \dot{x} &= 2x - y + 8e^{-t} \\ \dot{y} &= -x + 2y + 3 \end{aligned}$$

Find the general solution by the eigenvalue method (without reducing the system to a second-order differential equation).

4. Given is the economic model

$$\begin{aligned} \dot{K} &= 3\sqrt{K} - C \\ \dot{C} &= 8C - \frac{1}{2}KC - C^2 \end{aligned}$$

(a) Determine the nullclines and the equilibrium point (K^*, C^*) with $K^* > 0, C^* > 0$ by **computation**.

(b) Graph the nullclines and the sectors resulting from them and insert the directions of motions for all sectors.

5. Consider the following problem:

$$\min \int_0^1 (x^2 + tx + tx\dot{x} + \dot{x}^2)\, dt$$

$$x(0) = 0; \quad x(1) = 1$$

(a) Determine a solution $x = x(t)$ of the necessary optimality conditions **without** computing the constants (i.e., without using the initial and terminal conditions).

(b) Check whether the solution found in (a) is indeed a solution of the problem.