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This is a **self-test** of 30 multiple choice questions related to the above book. For any question, there exists exactly **one** correct answer of the five variants offered. One can find the correct answers and the number of obtainable points under the link

http://www.math.uni-magdeburg.de/∼werner/mct-answers.pdf

It is recommended to complete the test within no more than **100 minutes**.

MULTIPLE CHOICE QUESTIONS

Question 1: Given are the intervals $I_1 = [2,5], I_2 = [1,3], I_3 = (4,7)$. Which is the interval $I = (I_1 \setminus I_2) \cup I_3$?

Answer 1: $I = [3,7]$.

Answer 2: $I = [2,7]$.

Answer 3: $I = (3,7)$.

Answer 4: $I = [2,3] \cup (4,7)$.

Answer 5: $I = [1,2] \cup (4,7)$.

Question 2: What is the result of simplifying the term

$$T = 3[(a-b)(a+b)+b^2]^2 + [(a-b)^2 + 2ab]^2\,?$$

Answer 1: $T = (a+b)^4$.

Answer 2: $T = 4a^4 + 2a^2b^2 + b^4$.

Answer 3: $T = a^2(a^2+1) + b^2(b^2+1)$.

Answer 4: $T = 2(2a^2 + a^2b^2 + b^4)$.

Answer 5: $T = a^4 + b^4$.

Question 3: Which of the simplifications of the term $T = \frac{a}{a^2-2ab+b^2} - \frac{a}{a^2-b^2} + \frac{1}{a+b}$ is correct?

Answer 1: $T = \frac{a+b}{(a-b)^2}$.

Answer 2: $T = \frac{1}{a+b}$.

Answer 3: $T = \frac{(a+b)^2}{(a-b)^2}$.

Answer 4: $T = \frac{a^2+b^2}{(a-b)^2(a+b)}$.

Answer 5: $T = \frac{a+b}{a^2-b^2}$.

Question 4: Which real numbers are solutions of the equation $2\lg(x-1) = \lg(x+5)$?

Answer 1: There is no real solution of the equation.

Answer 2: The only real solution is $x = 4$.

Answer 3: The equation has exactly the two solutions $x_1 = 4$ and $x_2 = -1$.

Answer 4: The only real solution is $x = -1$.

Answer 5: The equation has more than two real solutions.

Question 5: Which real numbers are solutions of the equation $\left(\frac{4}{5}\right)^{x-7} = \left(\frac{25}{16}\right)^{x+3}$?

Answer 1: There exists only one real solution $x = \frac{1}{3}$.

Answer 2: There exists only one real solution $x = -10$.

Answer 3: There exist exactly two real solutions.

Answer 4: There exist more than two real solutions.

Answer 5: There exists no real solution.

Question 6: Which of the following equalities is not correct?

Answer 1: $(a^2 - b^2) = (a-b)(a+b)$.

Answer 2: $e^{x+y} = e^x \cdot e^y$.

Answer 3: $\log_2 x^y = y \log_2 x$.

Answer 4: $\ln(u+v) = \ln u + \ln v$.

Answer 5: $(\sqrt{x} - \sqrt{y})^2 = x + y - 2\sqrt{xy}$.

Question 7: Which real numbers satisfy the equation $3 + 2e^{-2x} - 5e^{-x} = 0$?

Answer 1: The only solutions of the equation are $x_1 = 0$ and $x_2 = -\ln\frac{3}{2} = \ln\frac{2}{3}$.

Answer 2: The only solution of the equation is $x = 0$.

Answer 3: There exists a real double solution $x_1 = x_2 = 0$.

Answer 4: The equation has no real solution.

Answer 5: The equation has the two solutions $x_1 = 0$ and $x_2 = 1$.

Question 8: It is known that a car consumes 6.2 l gas per 100 km. Which of the following equalities is not correct if x denotes how many km the car can go if the tank holds 40 l?

Answer 1: $6.2 : 40 = 100 : x$.

Answer 2: $6.2 : x = 100 : 40$.

Answer 3: $6.2 \cdot x = 100 \cdot 40$.

Answer 4: $6.2 : 100 = 40 : x$.

Answer 5: $x = \dfrac{4,000}{6.2}$.

Question 9: What can be said about the set L of real numbers satisfying the inequality $-4 > 2x - 2x^2$?

Answer 1: $L = (-\infty, -1) \cup (2, \infty)$.

Answer 2: $L = (-1, 2)$.

Answer 3: $L = \mathbb{R}$.

Answer 4: $L = (-\infty, -1] \cup [2, \infty)$.

Answer 5: $L = \emptyset$.

Question 10: What can be said about the (minimal) number of cases to be considered and the set L of real numbers satisfying the inequality $\frac{6}{2x+4} \leq |x|$?

Answer 1: We have to consider only one case and $L = [1, \infty)$.

Answer 2: We have to consider two cases and $L = (1, \infty)$.

Answer 3: We have to consider three cases and $L = (-\infty, -2) \cup [1, \infty)$.

Answer 4: We have to consider four cases and $L = \emptyset$.

Answer 5: We have to consider four cases and $L = (-\infty, -2)$.

Question 11: Which of the following equalities is not correct?

Answer 1: $\cos\alpha = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}$.

Answer 2: $\tan 2\alpha = \dfrac{1}{\tan\alpha - \cot\alpha}$.

Answer 3: $\sin 2\alpha = \dfrac{2\tan\alpha}{1+\tan^2\alpha}$.

Answer 4: $\cos^2\alpha = \dfrac{\cot^2\alpha}{1+\cot^2\alpha}$.

Answer 5: $\sin(\alpha + t) = \sin\alpha\cos t + \cos\alpha\sin t$.

Question 12: Which angles in the interval $[0, 360^o]$ are a solution of the equation $2\sin x + \cos x = 2$ (angles rounded to two decimal places)?

Answer 1: There are exactly the three solutions $x_1 = 36.87^o, x_2 = 90^o, x_3 = 143.13^o$.

Answer 2: There is exactly one solution $x = 90^o$.

Answer 3: There are exactly two solutions $x_1 = 90^o$ and $x_2 = 143.13^o$.

Answer 4: There are exactly two solutions $x_1 = 36.87^o$ and $x_2 = 90^o$.

Answer 5: There are the three solutions $x_1 = 36.87^o, x_2 = 90^o, x_3 = 143.13^o$, but there exist further solutions.

Question 13: What can be said about the curve of second order $4x^2 + y^2 - 8x + 4y = 8$?

Answer 1: It is a circle with the radius $r = 4$.

Answer 2: It is an ellipse with the half axes $a = 2$ and $b = 4$.

Answer 3: It is an ellipse with the half axes $a = 4$ and $b = 16$.

Answer 4: It is a hyperbola with the half axes $a = 2$ and $b = 4$.

Answer 5: It is a hyperbola with the half axes $a = 4$ and $b = 16$.

Question 14: Given is a sequence with the terms $a_1 = 1, a_2 = 1, a_3 = 2, a_4 = 3, a_5 = 5, a_6 = 8$. What are the terms a_7 and a_8 ?

Answer 1: $a_7 = 11,\ a_8 = 15$.

Answer 2: $a_7 = 12,\ a_8 = 17$.

Answer 3: $a_7 = 12,\ a_8 = 18$.

Answer 4: $a_7 = 13,\ a_8 = 19$.

Answer 5: $a_7 = 13,\ a_8 = 21$.

Question 15: A company starts with the production of 10,000 trucks in 2011. If the production increases every year by 10 %, what is the production in 2020 and what will be the first year so that in total at least 80,000 trucks are produced?

Answer 1: The production in 2020 is 20,000 trucks and in 2017, the total production exceeds 80,000 for the first time.

Answer 2: The production in 2020 is 21,435 trucks and in 2017, the total production exceeds 80,000 for the first time.

Answer 3: The production in 2020 is 23,579 trucks and in 2017, the total production exceeds 80,000 for the first time.

Answer 4: The production in 2020 is 25,937 trucks and in 2016, the total production exceeds 80,000 for the first time.

Answer 5: The production in 2020 is 28,531 trucks and in 2015, the total production exceeds 80,000 for the first time.

Question 16: What can be said about function $f : D_f \to R_f$ with $f(x) = \sin x - \dfrac{1}{2}\sin 2x$?

Answer 1: Function f has only the zeroes $x_k = 2k\pi, k \in \mathbb{Z}$, is odd and periodic with a period of π.

Answer 2: Function f has only the zeroes $x_k = k\pi, k \in \mathbb{Z}$, is odd and periodic with a period of 2π.

Answer 3: Function f has only the zeroes $x_k = 2k\pi, k \in \mathbb{Z}$, is even and periodic with a period of 2π.

Answer 4: Function f has only the zeroes $x_k = k\pi, k \in \mathbb{Z}$, is even and periodic with a period of π.

Answer 5: Function f has only the zeroes $x_k = 2k\pi, k \in \mathbb{Z}$, is neither even nor odd and also not periodic.

Question 17: Given are the functions $f : [0, 1] \to R_f$ with $f(x) = 5x - 1$ and $g : [-1, 1] \to R_g$ with $g(x) = \sqrt{1 - x^2}$. What can be said about the compositions $f \circ g$ and $g \circ f$?

Answer 1: Both compositions $f \circ g$ and $g \circ f$ do not exist.

Answer 2: The composition $(f \circ g)(x) = 5\sqrt{1 - x^2} - 1$ exists, while the composition $g \circ f$ does not exist.

Answer 3: The composition $f \circ g$ does not exist, but the composition $(g \circ f)(x) = \sqrt{1 - (5x - 1)^2}$ exists.

Answer 4: The composition $f \circ g$ does not exist, but the composition $(g \circ f)(x) = 2 - 4x$ exists.

Answer 5: Both compositions $f \circ g$ and $g \circ f$ exist.

Question 18: Given is the function $f : D_f \to R_f$ with $f(x) = \sqrt{x+1} - 2$. What is the domain D_f, the inverse function f^{-1} of function f and the domain $D_{f^{-1}}$ of the inverse function?

Answer 1: $D_f = [-1, \infty), \quad f^{-1}(x) = (x+2)^2 - 1, \quad D_{f^{-1}} = [-2, \infty)$.

Answer 2: $D_f = [0, \infty), \quad f^{-1}(x) = (x+2)^2 - 1, \quad D_{f^{-1}} = \mathbb{R}$.

Answer 3: $D_f = [-1, \infty), \quad f^{-1}(x) = (x+1)^2 - 2, \quad D_{f^{-1}} = \mathbb{R}$.

Answer 4: $D_f = [-1, \infty), \quad f^{-1}(x) = \sqrt{x+2} - 1, \quad D_{f^{-1}} = \mathbb{R}$.

Answer 5: $D_f = [0, \infty), \quad f^{-1}(x) = (x+1)^2 - 2, \quad D_{f^{-1}} = \mathbb{R}$.

Question 19: What can be said about function $f : D_f \to R_f$ with $f(x) = \dfrac{\cos \frac{x}{3}}{(e^x - \sqrt{e})(x+2)}$?

Answer 1: Function f has the zeroes $x = \dfrac{\pi}{2} \pm k\pi, k \in \mathbb{Z}$, and the only pole at $x = -2$.

Answer 2: Function f has the zeroes $x = \dfrac{3}{2}\pi \pm k\pi, k \in \mathbb{Z}$, and the only two poles at $x_1 = \dfrac{1}{2}$ and $x_2 = -2$.

Answer 3: Function f has the zeroes $x = \dfrac{3}{2}\pi \pm 3k\pi, k \in \mathbb{Z}$, and the only two poles at $x_1 = \dfrac{1}{2}$ and $x_2 = -2$.

Answer 4: Function f has the zeroes $x = \dfrac{\pi}{2} \pm 3k\pi, k \in \mathbb{Z}$, and the only two poles at $x_1 = 1$ and $x_2 = -2$.

Answer 5: Function f has the zeroes $x = \dfrac{3}{2}\pi \pm 3k\pi, k \in \mathbb{Z}$, and the only two poles at $x_1 = 1$ and $x_2 = -2$.

Question 20: Given is the function $f : D_f \to R_f$ with $f(x) = \dfrac{2x - 50}{\sqrt{x} - 5}$ and let $L = \lim\limits_{x \to 25} f(x)$.

What can be said about continuity / discontinuity of the function at the point $x = 25$ and the limit L?

Answer 1: Function f is continuous at $x = 25$ and $L = 0$.

Answer 2: Function f is discontinuous at $x = 25$, and the limit L does not exist since the right- and left-side limits as x tends to 25 do not coincide.

Answer 3: Function f has a pole at $x = 25$ and $L = \infty$.

Answer 4: Function f has a gap at $x = 25$ and $L = 20$.

Answer 5: Function f has a jump at $x = 25$ and L does not exist.

Question 21: What can be said about function $f : D_f \to R_f$ with $f(x) = 1 - \dfrac{2e^x}{e^x + 1}$?

Answer 1: Function f has no zero and no extreme point, but an inflection point at $x = 0$.

Answer 2: Function f has no zero and no inflection point, but a local extreme point at $x = 0$.

Answer 3: Function f has a zero and a local extreme point at $x = 0$, but no inflection point at $x = 0$.

Answer 4: Function f has a zero and an inflection point at $x = 0$, but it has no local extreme point at all.

Answer 5: Function f has a zero, an extreme point and an inflection point at $x = 0$.

Question 22: What can be said about monotonicity as well as convexity / concavity of function $f : D_f \to R_f$ with $f(x) = \dfrac{3x - 3}{2x^3}$?

Answer 1: Function f is strictly increasing and strictly convex on the domain D_f.

Answer 2: Function f is strictly decreasing and strictly concave on the domain D_f.

Answer 3: Function f is strictly increasing on $\left(-\infty, \dfrac{3}{2}\right]$ as well as strictly decreasing on $\left[\dfrac{3}{2}, \infty\right)$ and strictly convex on $(-\infty, 2]$ as well as strictly concave on $[2, \infty)$.

Answer 4: Function f is strictly increasing on $(-\infty, 0)$ and strictly decreasing on $(0, \infty)$ as well as strictly convex on $(-\infty, 0)$ and strictly concave on $(0, \infty)$

Answer 5: Function f is strictly increasing on $(-\infty, 0)$ as well as on $\left(0, \dfrac{3}{2}\right]$ and strictly decreasing on $\left[\dfrac{3}{2}, \infty\right)$ as well as strictly convex on $(-\infty, 0)$, strictly concave on $(0, 2]$ as well as strictly convex on $[2, \infty)$.

Question 23: What can be said about the limits

$$L_1 = \lim_{x \to 0} \frac{e^{3x} + e^{-3x} - 2}{5x^2} \quad \text{and} \quad L_2 = \lim_{x \to 0} \frac{e^{3x} + e^{-3x} - 2}{5x^2 - 2x} \,?$$

Answer 1: $L_1 = 1.8$ and $L_2 = 0$.

Answer 2: $L_1 = L_2 = 0$.

Answer 3: $L_1 = 0$ and $L_2 = -\infty$.

Answer 4: $L_1 = \infty$ and $L_2 = -\infty$.

Answer 5: $L_1 = L_2 = \infty$.

Question 24: How can the integral $\int (x^2 + x) \cdot e^{\frac{x}{2}}\, dx$ be found?

Answer 1: Apply integration by substitution with $t = x^2 + x$.

Answer 2: Apply integration by substitution with $t = e^{\frac{x}{2}}$.

Answer 3: Apply once integration by parts, differentiating the term $e^{\frac{x}{2}}$ and integrating the term $x^2 + x$.

Answer 4: Apply twice integration by parts, differentiating each time the term $e^{\frac{x}{2}}$.

Answer 5: Apply twice integration by parts, integrating each time the term $e^{\frac{x}{2}}$.

Question 25: Which of the following computations of the area A enclosed by the function $f : D_f \to R_f$ with $f(x) = \sin 2x$ and the x-axis between $x_1 = 0$ and $x_2 = 2\pi$ is correct?

Answer 1: $A = 4 \int\limits_0^{\pi/2} \sin 2x dx$.

Answer 2: $A = \int\limits_0^{2\pi} \sin 2x dx$.

Answer 3: $A = \left| \int\limits_0^{2\pi} \sin 2x dx \right|$.

Answer 4: $A = \int\limits_0^{\pi} \sin 2x dx + \int\limits_{\pi}^{2\pi} \sin 2x dx$.

Answer 5: $A = \left| \int\limits_0^{\pi} \sin 2x dx \right| + \left| \int\limits_{\pi}^{2\pi} \sin 2x dx \right|$.

Question 26: Which of the following answers is correct for the vectors

$$\mathbf{a} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} -1 \\ 4 \\ -2 \end{pmatrix}?$$

Answer 1: The vectors $\mathbf{a}$ and $\mathbf{c}$ are orthogonal.

Answer 2: We have $\mathbf{b}^T \cdot \mathbf{c} = 7$.

Answer 3: We have $|\mathbf{a}| = 4$.

Answer 4: We have $2\mathbf{a} + 3\mathbf{b} = \mathbf{c}$.

Answer 5: Vector $\mathbf{a} + \mathbf{b}$ is orthogonal to vector $\mathbf{c}$.

Question 27: How often does one need to throw a dice so that with probability P = 0.9, at least once the number 1 appears?

Answer 1: One has to throw the dice no more than 6 times.

Answer 2: One has to throw the dice 9 times.

Answer 3: One has to throw the dice less than 12 times.

Answer 4: One has to throw the dice 13 times.

Answer 5: One has to throw the dice at least 18 times.

Question 28: In an urn, there are 16 balls, each of them containing one of the numbers 1, 2, ..., 16. If one notices 5 different numbers and then draws 5 balls from the urn, how large is the probability P_1 that exactly 3 of the noticed numbers are drawn and how large is the probability P_2 that at most two of the noticed numbers are drawn (probabilities rounded to two decimal places)?

Answer 1: $P_1 = 0.13$ and $P_2 = 0.86$.

Answer 2: $P_1 = 0.38$ and $P_2 = 0.98$.

Answer 3: $P_1 = 0.01$ and $P_2 = 0.48$.

Answer 4: $P_1 = 0.48$ and $P_2 = 0.99$.

Answer 5: $P_1 = 0.11$ and $P_2 = 0.38$.

Question 29: Let X be a discrete random variable with the probabilities

$$P(X = 1) = 0.12,\ P(X = 2) = 0.25,\ P(X = 3) = 0.45,\ P(X = 4) = 0.15,\ P(X = 5) = 0.03.$$

How large are the probabilities $P_1 = P(X < 4)$, $P_2 = P(X \geq 3)$, $P_3 = P(2 < X \leq 4)$?

Answer 1: $P_1 = 0.98$, $P_2 = 0.63$, $P_3 = 0.6$.

Answer 2: $P_1 = 0.82$, $P_2 = 0.18$, $P_3 = 0.85$.

Answer 3: $P_1 = 0.82$, $P_2 = 0.18$, $P_3 = 0.45$.

Answer 4: $P_1 = 0.82$, $P_2 = 0.63$, $P_3 = 0.6$.

Answer 5: $P_1 = 0.98$, $P_2 = 0.63$, $P_3 = 0.7$.

Question 30: How can the probability that an $N(\mu, \sigma^2) = N(50, 4)$-distributed random variable is in the interval $[42, 62]$ be calculated using the distribution function Φ of the standard normal distribution?

Answer 1: One has to calculate $\Phi(62) - \Phi(42)$.

Answer 2: One has to calculate $\Phi(62) + \Phi(42) - 1$.

Answer 3: One has to calculate $\Phi(6) + \Phi(4) - 1$.

Answer 4: One has to calculate $\Phi(3) + \Phi(2) - 1$.

Answer 5: One has to calculate $\Phi(6) + 1 - \Phi(4)$.