

Exercises 'Mathematical Economics'

Series 1

- Investigate the definiteness of the following quadratic forms  $Q(\mathbf{x}) = \mathbf{x}^T A \mathbf{x}$ :
  - $Q(x_1, x_2, x_3) = 5x_1^2 + 2x_1x_3 + 2x_2^2 + 2x_2x_3 + 4x_3^2$ ;
  - $Q(x_1, x_2) = -(x_1 - x_2)^2$ ;
  - $Q(x_1, x_2, x_3) = -3x_1^2 + 2x_1x_2 - x_2^2 + 4x_2x_3 - 8x_3^2$ .
- Determine the gradients of functions  $f$  and  $g$  at the given points:
  - $f(x, y) = xy + y^2$  at  $(x^0, y^0) = (2, 1)$ ;
  - $g(x, y, z) = xe^{xy} - z^2$  at  $(x^0, y^0, z^0) = (0, 0, 1)$ .
- Determine the directional derivatives of functions  $f$  and  $g$  at the given points in the given direction:
  - $f(x, y) = 2x + y - 1$  at  $(x_0, y_0) = (2, 1)$  in the direction  $\mathbf{r}^1 = (1, 1)^T$ ;
  - $g(x, y, z) = xe^{xy} - z^2 - xy$  at  $(x_0, y_0, z_0) = (0, 1, 1)$  in the direction  $\mathbf{r}^2 = (1, 1, 1)^T$ .

4. → **Homework** ☞

Given is the function  $f : D_f \rightarrow \mathbb{R}$  with

$$f(x, y, z) = xy \ln(x^2 + y^2 + z^2).$$

- Find the directional derivative of  $f$  at the point  $(x_0, y_0, z_0) = (1, 1, 1)$  in the direction given by the vector from point  $(3, 2, 1)$  to point  $(-1, 1, 2)$ .
  - Determine the direction of maximal increase from point  $(x_0, y_0, z_0) = (1, 1, 1)$ .
- Find the quadratic approximations for the following functions at the point  $(0, 0)$ :
    - $f(x, y) = e^{x+y}(xy - 1)$ ;
    - $g(x, y) = e^{xe^y}$ ; → **Homework** ☞
    - $h(x, y) = \ln(1 + x^2 + y^2)$ .

- Assume that point  $(x_0, y_0; u_0, v_0)$  satisfies the two equations

$$\begin{aligned} F(x, y; u, v) &= x^2 - y^2 + uv - v^2 + 3 = 0 \\ G(x, y; u, v) &= x + y^2 + u^2 + uv - 2 = 0 \end{aligned}$$

Give sufficient conditions for this system to be represented by two equations

$$u = f(x, y), \quad v = g(x, y)$$

in a neighborhood of this point. Show that this condition is satisfied for

$$(x_0, y_0; u_0, v_0) = (2, 1; -1, 2).$$

(a) Compute  $f_x(2, 1)$  and  $g_x(2, 1)$ .

(b) Compute  $f_y(2, 1)$  and  $g_y(2, 1)$ . → **Homework**

