Faculty of Mathematics
Institute of Mathematical Optimization
Prof. Dr. F. Werner

<u>Exercises 'Methods for Economists'</u>

Series 3

1. Consider the function $g : D_g \to \mathbb{R}$ with $D_g = \{(x, y) \in \mathbb{R}^2 \mid x > 0, y > 0\}$ given by

$$g(x, y) = x^3 + y^3 - 3x - 2y.$$

Show that function g is convex and determine its global minimum value.

2. Let function $f : \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x, y) = x^3 + y^3 - 3xy.$$

Determine all local extreme points of function f.

3. Let $f : \mathbb{R}^3 \to \mathbb{R}$ and consider the problem:

$$\begin{array}{rcl} f(x_1, x_2, x_3) & = & 4x_1 + 5x_2 + 3x_3 \to \max! \\ \text{subject to} & & 2x_1^2 + x_2^2 + 3x_3^2 = 36 \end{array}$$

Solve this problem by using the Lagrange multiplier method and checking the necessary and sufficient conditions.

4. Let $f : \mathbb{R}^3 \to \mathbb{R}$ and consider the problem:

$$\begin{array}{rcl} f(x_1, x_2, x_3) & = & 5x_1 + 3x_2 - x_3 \to \max! \\ \text{subject to} & & x_1^2 + x_2^2 + x_3^2 = 72 \\ & & 4x_1 - x_2 = 0 \end{array}$$

Apply the Lagrange multiplier method (i.e. do **not** reduce the problem to one with only two variables and one constraint).

(a) Determine all points which fulfill the necessary optimality condition.
$\to$ **Homework**

(b) Determine the optimal solution of the problem by checking the sufficient optimality condition.

5. Consider the following cost minimization problem. Let

$$f(x, y) = x^{\alpha} y^{\beta} \qquad (\alpha, \beta > 0,\ \alpha + \beta = 1)$$

denote the production function and

$$c(x, y) = ax + by$$

the cost associated with the use of the two production factors x and y. Finally, let w denote the required output level.

(a) Set up the corresponding optimization problem and the Lagrangian function.

(b) Solve the problem in the usual way.

(c) Give an economic interpretation of the Lagrangian multiplier involved here.

6. Assume the following loss function for a central bank:

$$\mathcal{L}(U, \pi) = \frac{U^2}{2} + \theta \frac{\pi^2}{2}, \quad \theta = 0.1,$$

which shall be minimized subject to the following (short-run) Phillips-curve:

$$\pi = (-\beta_w)(U - \overline{U}) + \pi^e$$

with π denoting the actual rate of inflation, π^e the expected one, U the unemployment rate and $\overline{U}$ the 'natural level' of the latter. Assume that $\overline{U} = 7\% = 0.07$ and $\beta_w = 2$.

(a) Determine the short-run optimum with regard to U and π for $\pi^e = 0$.

(b) Give a (qualitative) graphical representation of your results.