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Exercises 'Methods for Economists'

Series 7

1. Solve the following initial value problems:
   (a) $(1+t^3)\cdot \dot{x} = t^2 \cdot x, \qquad x(0)=2;$
   (b) $\dot{x} = 6t^2 \cdot x^2, \qquad x(3) = -\frac{1}{38}.$
   → **Homework** ✍

2. (a) Assume that the marginal prospensity to consume in an economy is given by $C'(Y) = 0.8$. Furthermore, for the current income $Y_0 = 1000$ the consumption level is $C(Y_0) = 900$. Which consumption level will be reached if income rises to $Y_1 = 1200$.
   (b) Now assume that aggregate demand Y^d of an economy is composed of a consumption function $C(Y) = c\cdot Y,\ 0 < c < 1$, an autonomous investment $\overline{I}$ and a given level of government expenditure $\overline{G}$, i.e. $Y^d = C(Y) + \overline{I} + \overline{G}$. Assume furthermore that output reacts on the difference between demand and supply in the following way:
   $$\dot{Y}(t) = \mu\,[Y^d(t) - Y(t)] \ = \mu\,[C(Y) + \overline{I} + \overline{G}) - Y(t)], \quad \mu > 0.$$
   Now proceed in the following way:
      i. Determine the equilibrium state of this differential equation.
      ii. Find the general solution of the corresponding homogeneous equation.
      iii. Show that the sum of the solutions obtained in (i) and (ii) solves the original differential equation.

3. Find the general solution of the following differential equation:
   $$\dot{x} - \frac{t}{t^2-1}\cdot x = t, \qquad (t > 1).$$

4. Determine the equilibrium states and their stability properties of the following differential equation:
   $$\dot{x} = [7-x]^2 \cdot [x^2 - 6x + 5].$$

5. Solve the following initial value problems:
   (a) $\ddot{x} + 2\dot{x} + x = t^2, \qquad x(0) = 0,\ \dot{x}(0) = 1;$
   (b) $\ddot{x} + 4x = 4t + 1, \qquad x(\pi/2) = 0, \dot{x}(\pi/2) = 0.$